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Further information concerning the preparation of technical papers is contained in the "Technical Publications Handbook" which can be obtained from the Society.

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DESIGN STANDARDS FOR TIMBER STRUCTURES

Robert E. Eby,[1] A. M. ASCE

SYNOPSIS

Complete design standards permitting the engineer to design wood structures with the same efficiency as other materials have only become available during the past 15 years. This paper reviews the content and development of these design standards as well as supplemental data and standards pertaining to the use of wood as an engineering material.

INTRODUCTION

Wood was one of the first structural materials known to man and was used in all types of structures for centuries before the development of structural steel, reinforced concrete, aluminum and plastics. However, the fact that wood has been taken for granted—it has always been readily available, is easily worked with simple tools, and early structures were conveniently built by rule of thumb, no doubt served as a retardant in the development of timber design standards. On the other hand, the physical properties of the newer materials of construction required the determination of mechanical properties and development of engineering criteria from their beginning. The net result has required a concerted effort in recent years to restore timber as a major engineering material through the development of complete design standards.

Only in the past half century has there been extensive research and testing of the physical and mechanical properties of wood and the development of design criteria and standards. The non-isotropic properties of wood have presented more complications in the development of physical and mechanical properties than for other materials and has prolonged the development of condensed design data under one cover for convenient use by the structural engineer. While this desirable consolidation has yet to be fully accomplished, the

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goal is nearing attainment as only two standards are now required for the design of timber and plywood structures for the majority of uses and several excellent handbooks are available containing supplemental data for use as a test or for the practicing engineer's reference file.

There have been three significant factors resulting in the rebirth of wood as a major structural material. The first was the introduction of timber connecting devices in the early 1930's permitting more efficient design of mechanically connected joints. The second was the development of adhesives, particularly the synthetic resins in the late 1930's, which triggered the rapid growth of the plywood and glued laminated industries. The third was World War II, which accelerated the development of adhesives, design criteria and other factors related to wood as an engineering material.

The rapid growth of engineered wood in the past 15 years is a result of the combined effort of both government and private agencies and research laboratories, adhesive manufacturers, the wood preserving industry, the lumber and plywood associations and the timber fabricators. Committees of professional and trade societies such as ASCE, ASTM, AWPA and AREA have contributed to past developments and are important to future progress.

Timber Design Standards

National Design Specifications

"National Design Specifications for Stress-Grade Lumber and Its Fastenings" contains the pertinent design criteria necessary to design the majority of engineered wood structures, with the exception of plywood. The first edition of NDS was approved by the Board of Directors of the National Lumber Manufacturers Association in December, 1944 as the industry recommended design standard for post war construction. This edition was substantially the same as War Production Board Directive 29 and contained the same recommended stresses as were mandatory under Directive 29. The national emergency required the conservation of all materials and WPB Directive 29, issued in August, 1943 was a mandatory directive requiring a 20% increase in wood stresses over values formerly used. The 20% increase was not entirely arbitrary as a three year study by industry and government engineers of all available data actually preceeded the issuance of the directive.

A review of new data coupled with the excellent record of timber structures during the war and in post war construction resulted in final adoption of NDS in 1948 after another extensive review by industry and the Forest Products Laboratory. NDS is periodically revised to keep it up-to-date with new research and technical data.

Thus, it has only been 15 years that the timber engineer has had a complete design standard containing data and criteria based on research and testing programs and evaluated by laboratory, industry and practicing engineers.

NDS includes nine chapters plus an appendix covering subjects related to special application. Allowable Unit Stresses are tabulated for each stress grade of the 22 softwood and hardwood stress graded species as issued by eight regional lumber manufacturing associations. Design Loads and Design Formula and Provisions are covered in separate chapters. Mechanical fastenings are separated into four chapters covering Connectors, Bolts, Lag Screws and Nails, Spikes, Drift Bolt and Wood Screws. The necessary design criteria and unit stresses for Glued Laminated design are covered although not in as much detail as covered in the laminating standards of the regional associations.

AITC Timber Construction Standards

The American Institute of Timber Construction was founded in 1952 as the national, non-profit service organization of the engineered timber construction industry. The purpose of A.I.T.C. is to perform similar activities for this industry that the A.I.S.C. and A.C.I. perform for the steel and concrete industry. The first edition of the A.I.T.C. Timber Construction Standards was published in 1954 and revised in 1956.

The A.I.T.C. Standards cover a much broader scope than does NDS and includes a Code of Standard Practice, Fabrication Standards, Erection Standards as well as Design Specifications. The intent of the A.I.T.C. Standards is to cover subjects related to the entire field of engineered timber construction without duplicating other accepted standards. Therefore, the engineer will find information of general use but will not find specific design data such as unit stresses and design formula as the A.I.T.C. Standard refers to NDS and other Industry Standards for such data whenever applicable.

The Standards Committee of A.I.T.C. has developed several additional chapters for inclusion in the A.I.T.C. Standards and are currently working on new subjects, all of which will eventually appear in the A.I.T.C. Timber Construction Manual. Present thinking is that the A.I.T.C. Manual will include not only Design Standards but also general subjects pertaining to design, fabrication and erection of Engineered Timber Structures. It will be desirable to have such standards as NDS and Glued Laminated Specifications in the final manual so the architect and engineer will have all necessary data pertaining to timber design under one cover for ready reference.

New Standards that have been developed since 1954 include:

Standard Appearance Grades for Glued Laminated
Standard Protection for Glued Laminated
Trusses and Bracing
Guide Specifications for Glued Laminated
Significance of Checking—Glued and Sawn
Camber & Deflection—Glued Laminated
Selection of Adhesives

Other standards are being developed on the following:

Gluing of Treated, Treating of Glued and Sawn
Snow and Wind Loads
Typical Timber Construction Details
Heavy Timber Construction

The next year or two should see the completion of these and other projects and by 1960 a fairly complete A.I.T.C. Manual should be taking shape.

Plywood Design Standards

Technical Data on Plywood

The handbook "Technical Data on Plywood" is considered to be the accepted design standard on plywood. This Handbook is prepared by the Douglas Fir Plywood Association and is incorporated as a reference document by many building codes. The first edition of the handbook was printed in 1942 and since that time, new chapters have been added and data has been revised to keep up-to-date with research and development.

The Forest Products Laboratory developed and tested plywood stressed-skin panels in 1934 and published the first working stresses for plywood in 1941. In the past 15 years, the Douglas Fir Plywood Association has been the industry leader in research and the development of new applications of plywood as well as developing design criteria. The current edition of the Handbook contains chapters covering the General Design of Plywood, Deflection Charts, Lateral Bearing Strength of Nailed Plywood Joints, Form Factors of I and Box Beams with Plywood Webs, Design of Flat Panels with Stressed Covers and the Design of Built up Beams with Plywood Webs.

In addition to these chapters in the Handbook, the DFPA has published a pamphlet covering Plywood Sheathing and a recent brochure on Design of Plywood Diaphragms. No doubt these two subjects will eventually be included in the Handbook.

Grading rules, production requirements, sampling and testing, inspection and grade marking and certification of plywood are covered by U. S. Commercial Standards. Commercial Standards CS45 covering Douglas Fir Plywood was first adopted in 1933 and the latest revision, CS45-55 is included in the Handbook. Other Commercial Standards include CS-122 covering other western softwoods, CS157 covering western pine and CS35 covering hardwood plywood.

Supplementary Design Standards

There are several standards which are either basic to design data contained in NDS and Technical Data on Plywood, or are included all or in part in these design standards, or are considered supplemental standards not technically covering design.

Lumber Grading Rules

Specifications defining the strength reducing characteristics permitted and the assignment of allowable unit working stresses for structural grades might be considered the basic framework of all timber design. The grading rules for the various stress graded species are published by the eight regional lumber manufacturers association representing the 22 softwood and hardwood species that are currently stress graded.

The primary purpose of grading rules is to separate lumber into categories of nearly equal quality and appearance. The engineer is most concerned with the stress grades in which the main purpose is to select and classify lumber into groups on a strength basis so that unit working stresses can be assigned and used with confidence.

The first known grading rule is credited to Swan Averdson of Sweden in 1764. In 1833 the State of Maine passed a law recognizing four official grades which were similar to the earlier Swedish rules. The development of regional lumber manufacturers association in the late 1880's resulted in rules covering most commercial species by the early 1900's.

These early rules did not contain stress grades even though strength tests on wood are recorded in the "Memoirs of the French Academy of Science" as early as 1707 and by the English over 100 years ago. The earliest known tests recorded in this country were made by the Army Ordinance Department Watertown Arsenal in 1881. The U. S. Dept. of Agriculture initiated strength tests on wood in 1902 and in 1907 the American Society of Testing Materials

published its first Standard Specification for Structural Timber. The Forest Products Laboratory reported on the average ultimate values for two grades of ten species in 1912 and in 1923 an FPL circular described a method of stress grading and included working stresses for four standard grades correlated to 38 species. The system was refined, elaborated and republished in 1934 as U. S. Dept. of Agriculture Misc. Publication No. 185, and is basically unchanged today.

Unit stresses for all stress graded species are included in NDS so it is not necessary that the designer have copies of grading rules to be able to design with wood. However, familiarity with the provisions of the various grading rules is desirable and will enable the designer to better apply his judgment.

Glued Laminated Specifications

Four of the regional lumber manufacturers associations have published Glued Laminated Design and Fabrication Standards covering West Coast Douglas Fir, Southern Pine, Hardwoods and Western Larch. NDS contains the necessary design data and unit stresses required by the engineer but does not include the Fabrication section of the Regional Specifications.

The original design standard for glued laminated construction was Technical Bulletin No. 691 "The Glued Laminated Wooden Arch", prepared in 1939 by the Forest Products Laboratory. This Bulletin was revised to conform to new data and republished in 1954 as Technical Bulletin No. 1069 "Fabrication and Design of Glued Laminated Wood Structural Members".

The first industry glued laminated specification covered Douglas Fir and was published by the West Coast Lumbermen's Association in 1946. This standard was revised in early 1951 and was followed by the Southern Pine Standards in September - 1951, the Hardwood Standards in 1952 and by the Western Larch Specification in 1957. All of these standards conform to the technical requirements of Bulletin 1069 and have recommended stresses based on knot studies for each grade and combination of grades in accordance with the method of analysis contained in Bulletin 1069. The industry Glued Laminated Specifications have been generally accepted by architects, engineers, building codes and other specifying agencies.

American Lumber Standards

"American Lumber Standards for Softwood Lumber", published by the U. S. Dept. of Commerce is an important standard in that its development is responsible for eliminating the chaos that existed in the industry because of non-uniform size and classification of lumber. The early grading rules helped standardize within regions but each region had different sizes, grades, classifications, shipping and inspection practices. Only a few years ago, one species was known by twenty-nine different names, there were several methods of determining a board foot and the variation in thickness of one inch boards was more than 3/8 inches.

During World War I, the War Industries Board made a concentrated effort to conserve the natural resources by elimination of non-essential sizes, grades and types of manufactured products. The softwood industry followed through by organizing a program for simplification and standardization of lumber sizes and grades at an industry wide meeting of the American Lumber Congress in 1919. In 1921, private citizen Herbert Hoover, who was then

president of the Federated American Engineering Societies, appointed a committee of industrial management engineers to survey wasteful practices in six industries. This committee report of wasteful practices resulted in the establishment of the Division of Simplified Practice of the U. S. Dept. of Commerce in 1921 when Mr. Hoover was Secretary of Commerce. In 1922 the softwood lumber industry requested use of the facilities and procedures made available by Mr. Hoover. After many meetings, "Simplified Practice Recommendation No. 16-Lumber" better known as the "American Lumber Standards for Softwood Lumber" was published in 1924 and has been periodically revised and refined by a standing committee composed of representatives of producers, distributors and users of softwood lumber.

All softwood grading rules are written to conform to the classifications, sizes and other provisions of ALS and many codes and specifications require conformance to these standards. Hardwood grading rules conform to similar standards established by the hardwood industry.

American Society of Testing Materials Standards

There are several ASTM Standards on wood including Standards for testing small clear specimens, testing structural timbers, cyclic delamination tests for glued laminated, shear block tests for glued laminated and standards for piling. The standard of particular interest to the designer is ASTM D-245 covering "Standard Methods of Establishing Structural Grades of Lumber". This standard was first published as a tentative in 1926 and has been revised in 1927, 1929, 1930, 1933, 1936 and 1949.

A.S.T.M.D-245 covers the basic principles for establishing structural grades and includes the necessary procedures for the assignment of the various unit stresses to any grade of lumber. Included are tables of basic stresses, tables of strength ratios for strength reducing characteristics and modification for seasoning and duration of load.

All stress grades are based on this standard, the provisions of which are also included in the Wood Handbook and which are based on research by the Forest Products Laboratory. The basic principles of stress grading are to determine the strength of the clear wood for the various species, evaluate the factors that affect strength and apply these factors to structural grades on the basis of practically every individual timber being capable of safely carrying its full design load for the life of the structure. This requires consideration of many factors such as effect of knots, checks, slope of grain, variability, duration of load and condition of use. The application of these standards provides the most accurate determination of strength properties by visual means of any structural material resulting in general acceptance of industry recommended stresses by building codes and other specifying agencies.

AWPA Manual

The "Manual of Recommended Practice" published by the American Wood Preservers Association is the industry standard on pressure preservative treatments. This standard covers the chemical composition of the preservatives and fire retardant formulations, the treating process and recommended retentions for various classifications of use. The manual also includes miscellaneous standards such as methods of analysis, method for determining penetration, methods of sampling and standard instructions for inspection and purchase of preservative treated products.

The Service Bureau of the AWPA also publishes design manuals on specific subjects including several on the design of timber-concrete composite decks and a recent treatise on "How to Design Pole Type Buildings". These are special treatments of specific design problems and serve as excellent references for the engineer who might have occasion to design that type of structure.

AREA Manual

The American Railway Engineering Association "Manual of Recommended Practices" includes Chapter 7 on "Wood Bridges and Trestles". This railway design standard includes specifications for structural timber, glued laminated lumber and wood piles and specifications for the Design of Wood Bridges and Trestles for Railway Loading. Also included is a Construction and Maintenance Section and a Section of Typical Plans of Wood Railway Bridges.

AASHO Specifications

The American Association of State Highway Officials publication "Standard Specifications for Highway Bridges" includes design criteria for Timber and Glued Laminated Use in Highway Bridges. Provision for connectors, timber-concrete composite decks, wood piles and preservation are also included in this Highway Bridge Standard.

ASA Standards

The American Standards Association publishes a standard for the classification of wood poles. The pole species are placed in six groups based on ultimate bending strength and classes of poles are arranged so that all poles of the same class will have the same strength regardless of species.

Technical References

There are several excellent technical references on wood that the timber engineer should have in his files to supplement the design standards.

Wood Handbook

U. S. Department of Agriculture Handbook No. 72, better known as the Wood Handbook, is the basic reference on wood technology and is the source of most data from which Timber Standards are developed. For the most part the contents represent research by the Forest Products Laboratory and includes chapters on Structure of Wood, Physical Properties, Strength Values, Grades and Sizes, Stress Grades, Connectors and Fastenings, Glues and Gluing, Plywood, Sandwich Construction, Moisture Control, Fire Resistance, Painting and Finishing, Preservation, Poles and Piling, Insulation, Fiberboards and Modified Woods and Paper-Base Laminates. The Wood Handbook was revised and reprinted in 1955.

TECO Handbook

The "Timber Design and Construction Handbook" prepared by Timber Engineering Company and published in 1956 is the most complete and up-to-date text type reference on Timber Engineering. The three sections of this manual

cover Basic Properties, Design Considerations and Design Standards. Excellent illustrations and practical problems combined with text material drawn from experienced timber engineers make for easy understanding by the student or professional. Both NDS and the TECO Connector Manual are included as well as many tables of properties applicable to design.

Douglas Fir Use Book

Published by the West Coast Lumbermen's Association, the 1958 revised edition of the Douglas Fir Use Book includes structural data and design tables for use by students, architects and engineers. Several new chapters have been added including Glued Laminated, Lateral Forces and Fire Protection. About half of the book consists of useful tables for quick determination of required sizes.

Wood Structural Design Data

National Lumber Manufacturers Association publishes "Wood Structural Design Data" which is quite similar to the Douglas Fir Use Book—the principal difference being that WSDD span tables are set up for bending stresses from 800f to 2300f and E values from 800,000 to 1,600,000 whereas the Douglas Fir Use Book tables are based on Douglas Fir stress grades only. WSDD was revised and reprinted in 1956.

Modern Timber Engineering

The handbook "Modern Timber Engineering is written by Scofield and O'Brien and published by the Southern Pine Association. This is a text type book with many problem examples and does not include tables like the Use Book and WSDD. The scope of the book was broadened with the 1954 revised edition and is used by some colleges as the text for courses in Timber Design.

Timber Engineers Handbook

This handbook was edited by H. J. Hansen in cooperation with the Weyerhaeuser Sales Company's Technical Staff. It is a comprehensive book of some 900 pages of which about two-thirds is tables and one-third text covering pertinent phases of timber design. This handbook was published in 1948 and has not been revised.

TECO Connector Manual

The "Design Manual for TECO Timber Connector Construction" is prepared by Timber Engineering Company and is used by many engineers for connector design as the presentation of connector values by means of curves is more convenient than the tabular presentation in NDS. The TECO Manual was first published in 1939 and the latest revision is 1954.

The Standards and References discussed herein, while not all inclusive, include those applicable to the majority of design problems. For example, Standard Specifications on crossarms and similar speciality items have been purposely omitted. As a minimum requirement, the designer should have a copy of NDS and Technical Data on Plywood. Additional Standards and References will depend on the type of design work but he will probably next add Grading Rules, Glued Laminated Specifications, A.I.T.C. Standards, Wood Handbook and one of the Handbook References such as the Douglas Fir Use Book or TECO Handbook.

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STATISTICAL APPROACH TO WORKING STRESSES FOR LUMBER[a]

J. D. Snodgrass[1]

INTRODUCTION

Stress-graded lumber is a familiar item on many structural engineering projects. Where timber is not the principal construction material, much stress-graded lumber may be required for auxiliary structures, such as falsework or scaffolding, that must be carefully designed and detailed. Where timber is the principal building material, stress-graded lumber is in large demand, because its specification insures that expectations for structural design will be realized.

Stress grades of lumber fabricated into glued-laminated structural members also serve the engineer in important ways. In many instances, laminated rather than solid, one-piece members will best meet structural, architectural, or economic requirements. Special laminating grades of lumber have been developed to provide the fabricator a wide range of stress-rated stock that can be worked with high efficiency into various well-known elements such as beams, columns and arch ribs. Availability of lumber grades with this type of versatility greatly extends the range of possible applications of timber for engineered structures.

Whether as individual members or incorporated in glued composites, stress-graded lumber is important in the family of modern engineering materials.

A large number of lumber grades in many species have been called "nonstress grades", to distinguish them from grades perhaps more familiar to the engineer. Nonstress-grade material is dimension lumber from 2 to 4 inches thick, or small timbers intended for both permanent and temporary general construction. It is consumed in huge quantities, especially in light-frame structures such as residences and medium-size commercial buildings. Two-inch thick dimension lumber in various widths is probably in greatest demand for light framing.

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a. Presented at the June 1958 ASCE Convention in Portland, Oregon.

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In a sense, each stick of lumber irrespective of size, species, or grade has a "stress rate". That is to say, every piece has an ultimate strength in bending, shear, compression, or tension. If properly modified by the engineer, these ultimate strengths could be reduced to working stresses. It would be tedious and unrealistic, however, to establish some type of variable modification factor for every grade, species and new design problem encountered in timber engineering. A realistic approach, and the one now well-established in practice, is to develop a system of broad grading principles that can be applied in a generally uniform way by the various regional lumber inspection agencies, or others, to particular species and quality classes of structural lumber. Application of this general procedure has kept the heavy building industry supplied for many years with accurately stress-rated timber products.

As for nonstress grades, there seemed little need in the past for developing a system of structural grading similar to that established for stress-graded structural items. In recent times, however, demand has been expressed for detailed engineering data, usually in the form of working stresses or span tables, on light-framing lumber. This information has been placed in various building codes, inspection guides, Federal Housing Administration Minimum Property Standards, and similar reference specifications. The need for more specific engineering data on nonstress grades appears to follow the continuing trend toward a more rational, technical approach to specification and use of major materials.

If traditional criteria for determining allowed working stresses for stress grades of lumber are applied to nonstress grades, rather low working stresses result. This result is due to the fact that nonstress grades, while providing lumber amply strong for its customary uses, may permit inclusion of a rather wide and variable range of strength-reducing characteristics. This circumstance will make a given grade somewhat more variable in strength than results where the strength-reducing characteristics are more strictly limited, as they are in the stress grades. To alter nonstress grades would add to expense of such lumber and likely would bring into use material having vastly more strength than actually needed in many applications. It seems desirable, therefore, to explore possible new systems of determining working stresses for nonstress grades of lumber. This task should be approached by taking note of the strength characteristics of existing, popular commercial grades and of the ways these grades are used in construction.

In light of the above philosophy, a study was made of two typical nonstress grades of Douglas fir dimension lumber. Primary objective of the study was to determine bending, compression and tension strengths of Construction and Standard[2] 2- by 4-inch unseasoned lumber. It was planned to express the strength properties typical of each grade as distributions of strength among individual pieces. It was planned also to express the strength properties as

2. When this study was started, Number 1 and 2 Studding, Blocking and Small Posts, as described in paragraphs 208 and 209 of Standard Grading and Dressing Rules No. 14, West Coast Lumbermen's Association, were the study grades. Before work was completed, present West Coast Rules No. 15 were issued. Grades identical to study grades now are known as Construction and Standard Light Framing, as described in paragraphs 122-b and 122-c of Rules No. 15. The present grade names are used in this paper.

distributions of the averages of 3 pieces presumed to carry loads jointly. Secondary objectives were to appraise the influence upon strength of seasoning representative material, and to estimate the effect upon strength of slightly altering the composition of grades by minor modifications of grade specifications.

The primary objective was suggested by the West Coast Lumbermen's Association, and work was performed by personnel of the Forest Products Research Center.[3] The U. S. Forest Products Laboratory and West Coast Lumber Inspection Bureau provided much instrumental assistance.[4] The Research Center developed data on strength of two- by four-inch Douglas fir lumber as a basis for determining working stresses for grades studied. It was hoped, also, that experience with a relatively new approach to establishing working stresses for nonstress-graded lumber would provide stimulus for additional studies of this type and perhaps for evolution of a simplified standard procedure applicable to all species and grades.

Background

The approach used in the present study can best be appreciated by a brief review of technical progress leading to establishment of present stress-grading practices. Three main lines of progress are discernable in North American lumber technology. First, fairly complete inventories have been made of basic strength and elastic properties of important timber species. This task has been accomplished through systematic testing programs, primarily at government laboratories. Second, experimental evidence has been gathered on various factors that influence the strength of wood, and limited to ample information is available. The relationship of duration of load to strength, for example, has been firmly established. However, a number of more subtle, and perhaps more variable, influences require additional study, but progress is being made on these problems. Third, direct tests of large structural members have been performed in reasonable abundance in the United States, in Canada, and elsewhere. The general scope of structural timber tests performed at government laboratories in North America is suggested by the incomplete list in Table 1. More tests have been performed than are enumerated in the partial summary, but records are not always accessible.

Full-scale timber tests performed in the past have certain limitations in terms of providing more than general information, because both timber quality and characteristics of commercially available grades have changed to some extent over the years. This change complicates interpretation of past test results for application to present circumstances. Nonetheless, past large-scale timber tests, together with other information already described, served as the basis for U. S. Department of Agriculture Miscellaneous Publication 185, published in 1934.(10) This document and various succeeding ones(11,1,7)

3. Particular acknowledgement is due J. W. Johnson and A. D. Hofstrand who conducted field work and performed laboratory tests for the Center. Personnel of the Associations's Technical Service Department assisted during field work.
4. Cooperation by L. W. Wood and John T. Drow at the Laboratory was especially appreciated.

Table 1. Partial Summary of Full-Scale Tests of Sawed Beams, Joists, Posts, and Columns Made at North American Government Laboratories*.

Item	Moisture condition		Species	Size
	Unseasoned	Seasoned		
	Basis	Basis		Inches
United States				
Beams	700	400	Douglas fir, southern pines, eastern and western hemlock, western larch, tamarack, red pine and Sitka spruce.	8 by 16
Joists, planks	120	120	Southern pines, eastern hemlock, ponderosa pine and mountain Douglas fir.	2, 3,& 4 by 12; 4 by 6; 3 by 8
Columns, short	1,150	840	Similar to species for beams.	4 by 7 to 8 by 8; 12 by 12.
Columns, long	80	80	Douglas fir and southern pines.	12 by 12
Canada				
Joists, planks	280	3,460	Red, jack and white pines; white spruce, balsam fir, eastern hemlock, Douglas fir and western hemlock.	2 by 4 to 3 by 9 and 2 by 10.

* Sources: U. S. Department of Agriculture, Forest Service Bulletins 88 and 108, Technical Bulletin 167; U. S. Forest Products Mimeograph report R1780; correspondence. Canadian Forest Service Circular 54, Canadian Woods, and correspondence.

contain criteria generally known as the "stress-grading principles". As applied by regional lumber inspection agencies to grades developed in the various producing areas, there are only minor differences in interpretation of the principles. In general, the effect of Circular 185, and amplifications of its basic guides, has been to provide a sound basis for uniform and effective structural lumber grading. The essential procedures described in Publication 185 were brought together in A.S.T.M. Tentative Standard D245-49T, published in 1949 and revised in 1957.[1]

Two concepts in the stress-grading scheme are particularly useful. One is the "basic stress," which is a safe stress under full, long-time loading applicable to clear, straight-grained, full-size members. Basic stresses apply to the mythical 100 per cent grade, the straight-grained, defect-free piece of timber. The other concept is the "strength ratio" which by definition is:

$$\text{Strength ratio} = \frac{\text{Strength of full-size commercial timber}}{\text{Strength of clear, straight-grained wood from the timber}} \times 100$$

Strength ratios commonly are expressed in per cent, and usually both commercial timber and clear wood from it are assumed to be unseasoned. Strength ratios had an important function in the present study.

When the practice of laminating lumber into large structural items became common and economic practice, modified concepts were needed for studying strength of structural timber and deriving working stresses. At this stage of technical development, mostly during and after World War II, the notion developed of a statistical approach for answering questions about adequacy of a timber's strength. It was evident that two important aspects favored higher working stresses for laminated structural members than for one-piece, non-laminated timbers of comparable quality. First, lumber for laminating had to be reasonably dry for component parts to be glued satisfactorily.

Uniform and adequate dryness, which could not be justified for large, solid, sawed members, gives a favorable strength increase. For example, increases of 25 per cent in bending strength were recognized in the case of glued, laminated members.[5,7] Second, the chance combination of weak with strong pieces through glue-laminating results in a composite piece stronger than if variable-strength individual members were used singly. The advantage in terms of increased average strength was recognized for random combination of thin individual pieces bonded into a large member.

After much research, the U. S. Forest Products Laboratory developed general procedures for establishing working stresses for laminated lumber members. These procedures incorporated a statistical approach to derivation of working stresses and are outlined in U. S. Department of Agriculture Technical Bulletin No. 1069, published in 1954.[5]

Associated with developments in design criteria for glued, laminated lumber construction were re-evaluations of the factor of safety of one-piece structural members. Results of this work were published in 1957.[15] They were useful extensions of previous studies on the variation of strength of wood used for structural purposes.[14] These studies took into consideration the various factors affecting strength of timber in place. A statistical approach was suggested for determining a distribution of the "factor of adequacy" of individual wood members. Pieces were assumed to function under a number of influences, each expressed as a frequency distribution. These statistical experiments were stimulated by thinking among engineers about probability principles used as an engineering tool. In the present study, the fruits of recent analysis by statistical methods have been applied to the simple, but practical, problem of forecasting strength of dimension lumber.

General Approach

The theory here applied was that all three components of the strength-ratio formula are best expressed as frequency distributions and that proper manipulation of 2 of the 3 distributions should lead to an expression for the third. The components are strength ratio, strength of clear wood, and strength of full-size structural member. Of these components, strength ratio (SR) and clear-wood strength can be, or already have been, measured; a sufficiently large number of such measurements can be expressed as distributions.

Strength of full-size pieces of lumber also could be measured, directly and accurately. A statistical approach, however may avoid the costly step of machine testing many full-size specimens, although the advantages of doing so are obvious. The strength of full-size pieces therefore was taken as the unknown and was computed through manipulation of the easily measurable elements, strength ratio and clear-wood strength. The previously mentioned strength-ratio equation was rearranged as follows:

Strength of full-size piece = Strength ratio X Clear-wood strength
(as a distribution) (each expressed as distributions)

It is necessary to know or assume that measured components are independent variables; it is further necessary to plan a large number of observations on elements selected for expression as distributions.

Strength ratios were estimated for individual pieces of a large randomly selected sample from each of the two study grades, Construction and Standard. These estimates then were expressed as frequency distributions of SR (strength ratio). Clear-wood strengths also were expressed as frequency distributions. The sources of individual clear-wood strength values varied depending upon abundance of available information.

A large number of random products of SR times clear-wood strength were prepared, and the random products were expressed as frequency distributions. These distributions were taken to be accurate estimates of strength under short-time (about 5 minutes) load conditions of single pieces of lumber from the respective study grades.

If the concepts followed are valid, machine testing of a large random sample from the same grades should result in identical distributions of the strength property for short-time load duration.

A further step of theory was recognition of the concept that in practical construction with nonstress-graded items of lumber, such as pieces 2 by 4 inches in size, chances are extremely remote that each piece will be required to carry a full-capacity load for a long time. Light frame construction is so composed that groups of associated members jointly carry many of the loads. It seemed reasonable, therefore, to suppose that an average design stress of some type might be justified where groups of members mutually share load-carrying functions. The situation recognized is roughly analogous to the joint functioning of individual variable-strength laminae in glued laminated timbers as they provide composite resistance to external forces.

Data are somewhat meager that demonstrate how forces in light frame construction are distributed among members. Solution to this problem is made more difficult by many uncertainties about direction and magnitude of applied forces. For the present study, however, it was desired to express frequency distributions of strength for groups of members jointly carrying impressed loads. Experimental basis existed to show that 3 members of a conventional floor-joist system shared in carrying spot loading on one member, because of force distribution laterally through the floor system. A group of 3 members, therefore, was assumed for computations in the present study. Distributions of the average of 3 randomly selected strength values from the basic product distributions were obtained by simple mathematical transformations of product distribution data.

Details of Procedure

Experimental procedure followed 4 main steps, (a) estimation in the field of strength ratios of randomly selected pieces of lumber in the 2 study grades, (b) check tests of bending strength on a one-fifth sample randomly drawn from the large sample, (c) assembly of basic data on both strength ratios and clear-wood strengths in a form suitable for computations, and (d) computations.

Three strength properties, bending, compression and tension (both parallel with the grain) were analyzed to accomplish the main objective. Strength distributions were developed for single pieces and averages of 3 pieces. Basic data for the principal computations then were modified slightly, or were supplemented as required for a preliminary study of the influence of seasoning on bending strength, and for a brief analysis of grade modifications as related to distributions of strength.

Estimating Strength Ratios

Ten pieces in each of 2 length classes, 8- and 12-foot, in each study grade were selected randomly at each of 10 sawmills in the Douglas fir producing areas of western Oregon, Washington and California. Pieces were selected from unseasoned, previously grade-marked stock. Each piece was assigned a strength ratio according to guides of ASTM Tentative Standard D245.(1) Where the ASTM Standard was not explicit, supplemental guides developed cooperatively with the U. S. Forest Products Laboratory were used. The sample inspected included 4 grade-length groups with 10 specimens in each group at 10 sawmills, or 400 pieces. Actually, a few more than 400 pieces were inspected, since additional randomly selected pieces were included for special reasons.

Check Tests

One-fifth of the field sample, about 80 pieces, was brought to the Center for machine testing in static bending under third-point loading (A.S.T.M. Standard D198-27).(3) Purpose of the tests was to establish the accuracy of field estimating, because true strength ratios of pieces tested could be determined in the laboratory. Modulus of rupture of structural-size pieces and of 2 or more standard, small clear specimens cut from the large pieces, specific gravities, and moisture contents were obtained from the series of check tests. True strength ratios for each piece of the one-fifth sample were computed from the usual strength-ratio formula, except that the moisture basis for the large pieces was air-dry (about 12 per cent moisture content) and for the small clears, green.

Data for Computations

Individual values for the two components, SR and clear-wood strength, were required for the computation step. Individual values for clear-wood strengths were obtained from 3 sources:

(a) Individual test values from the clear, unseasoned, minor test specimens broken during the check testing program were considered the most appropriate for describing distribution of clear-wood bending strength. Although only 160 individual values of unseasoned-basis modulus of

rupture were available, it was decided these values would best describe clear-wood bending strength of the population actually sampled in the field.

(b) Individual values of maximum crushing strength parallel to grain for unseasoned Douglas fir were not gathered during the study, but a large number of representative values was obtained from the U. S. Forest Products Laboratory. These data were made available on punched cards. About 4500 values from standard compression tests of Douglas fir collected in Northwest United States were supplied to the Center.

(c) Individual values for ultimate tensile strength parallel to grain for Douglas fir were gathered by Oregon State College, Division of Forest Research, working cooperatively with the Center.(8) The study of tensile strength was completed in time for results to be of value in the present analysis. About 260 individual values for unseasoned, and 270 values for dry (12 per cent moisture content) tensile strength of Douglas fir were available. Only values from unseasoned pieces were used in the present study. Previous standard tests(6) of tensile strength of Douglas fir were limited. The project referred to above, therefore, brings out new and extended information on tensile strength.

Computations

Main computations involved formation of random products of SR X clear-wood strength for each property studied. Total number of products made depended upon the amount of input data available and hence varied with each property. Total products formed ranged from about 1,000 to 3,000 for each property and grade.

Previous reports(13) suggested conversion of SR and clear-wood strength data to normal frequency distributions. These normal distributions were manipulated during formation of random products. In the present study, input data were not converted to normal distributions, but were used in a "raw" state as individual values.

A digital computer functioning from punched cards carrying input information was programmed to make random products, punch the products into more cards, and tabulate the products as cumulative distributions of products classified into appropriate stress-range groups. Tabulated outputs of cumulative distributions (card counts) were plotted on probability graph paper for further study.

For the preliminary study of influence of seasoning on bending strength of dimension lumber, a third distribution of individual values was combined randomly with distributions of bending SR and clear-wood moduli of rupture by procedures similar to those already described. The new distribution used at this step defined the relative strength increase caused when clear green wood is seasoned. Individual strength values from which the relative increases were computed were obtained from tests of about 130 matched pairs of small bending-test sticks cut from residue study material. A specimen of each pair was tested unseasoned, the other tested air-dry at about 12 per cent moisture content. The computations of random products involved manipulation of three distributions as follows:

Estimated dry strength = SR (green basis) X Clear-wood strength (green) X
of full-size pieces Relative strength increase
(as a distribution) (each as a distribution)

Analyses for the compression and tension properties could be made following the procedure outlined above. Such analyses were not made during the present study partly because adequate data were not readily available, but primarily because the analysis on bending was purely tentative and experimental.

There is logical objection to the procedure described above for estimating strength of dry, full-size members from data applicable to full-size, unseasoned members. The suggested procedure fails to consider possibly important offsets against gains in strength by clear wood as it seasons. Such offsets are thought to occur because, as certain full-size structural pieces dry, their natural strength-reducing characteristics may become relatively more serious. Therefore, net gains in strength due to seasoning may or may not be realized in performance of full-size pieces. This objection was recognized fully when the analysis described above was planned. The experiment was performed anyway to satisfy curiosity about the result of incorporating an expression for increase due to seasoning into the random product computations. Presently described results of this trial should be considered in light of this qualification.

Similar to the "seasoning increase" analysis, study of the effect of arbitrarily altering the grades of study material was made to serve a limited, secondary objective. Results may not have practical significance at this time, because the effect of modifying grades purely for experimental purposes is to set up an arbitrary nonexistent grade. The analysis involved removal from the SR component of all values below 38 per cent and re-computation of bending-strength distributions for the grades thus artificially established. Removal of certain strength ratios had the effect, in a majority of instances, of reducing the size of the largest knot permitted in the respective grades. Under these conditions, strength distributions of the grades should change. The experiment was performed to learn how much change could be expected.

Results and Discussion

Strength-Ratio Estimates

Characteristics of the distributions of estimated strength ratios for Construction and Standard Douglas fir 2- by 4-inch dimension lumber, unseasoned, are given in Table 2 and illustrated in Fig. 1. The two lumber-length classes within each grade are given separately in Table 2 but were combined in Fig. 1. Length classes were pooled for all computational operations, when it was observed that there were only minor differences in means and standard deviations between the 8- and 12-foot length groups within grades. Means and range of strength ratios in each grade are worthy of note (Table 2). The ranges of SR apparently are broad for the grades studied and vary among the 3 major strength properties. Some departures of SR distributions from normality are evident in Fig. 1.

Clear-Wood Strength Distributions

Some of the characteristics of distributions for clear-wood strength in bending, compression and tension are summarized in Table 3. Distributions are illustrated in Figs 2, 3 and 4. Straight lines on these arithmetic probability charts define normal distribution of individual values about their mean.

Table 2. Basic Data from Estimates of Bending, Compression, and Tension Strength Ratios of Construction and Standard Grades, Unseasoned Douglas fir 2- by 4-inch Dimension Lumber at 10 Sawmills in Western Oregon, Washington, and California.

Strength ratio		Bending				Compression				Tension			
		Construction		Standard		Construction		Standard		Construction		Standard	
Classes	Value	8-foot	12-foot	8-foot	12-foot	8-foot	12-foot	8-foot	12-foot	8-foot	12-foot	8-foot	12-foot
	Per cent						Number of estimates						
1	10.0-19.9	-	-	1	4	-	-	-	-	-	-	2	-
2	20.0-29.9	2	3	7	8	-	-	1	1	-	1	1	1
3	30.0-39.9	1	7	14	10	-	-	3	2	1	-	5	8
4	40.0-49.9	9	6	20	18	-	1	6	3	4	7	12	7
5	50.0-59.9	30	27	29	37	9	7	20	20	8	19	24	26
6	60.0-69.9	24	29	17	14	14	27	33	31	25	27	36	28
7	70.0-79.9	20	15	10	8	44	43	29	30	39	29	14	20
8	80.0-89.9	10	9	4	2	29	19	13	14	20	13	11	11
9	90.0-99.9	10	6	5	0	10	5	2	-	9	6	2	-
	Totals	106	102	107	101	106	102	107	101	106	102	107	101
	Means	65.9	62.2	53.5	50.0	75.6	72.2	64.7	65.4	71.8	67.6	60.1	60.8
Std. deviations		16.5	16.4	17.8	15.5	10.7	10.5	13.1	11.4	12.5	13.7	14.7	12.8
Means for 8- and 12-foot combined		64.0		51.7		73.9		65.1		69.7		60.4	
Standard grade mean as fraction of Construction grade mean		0.82				0.88				0.87			

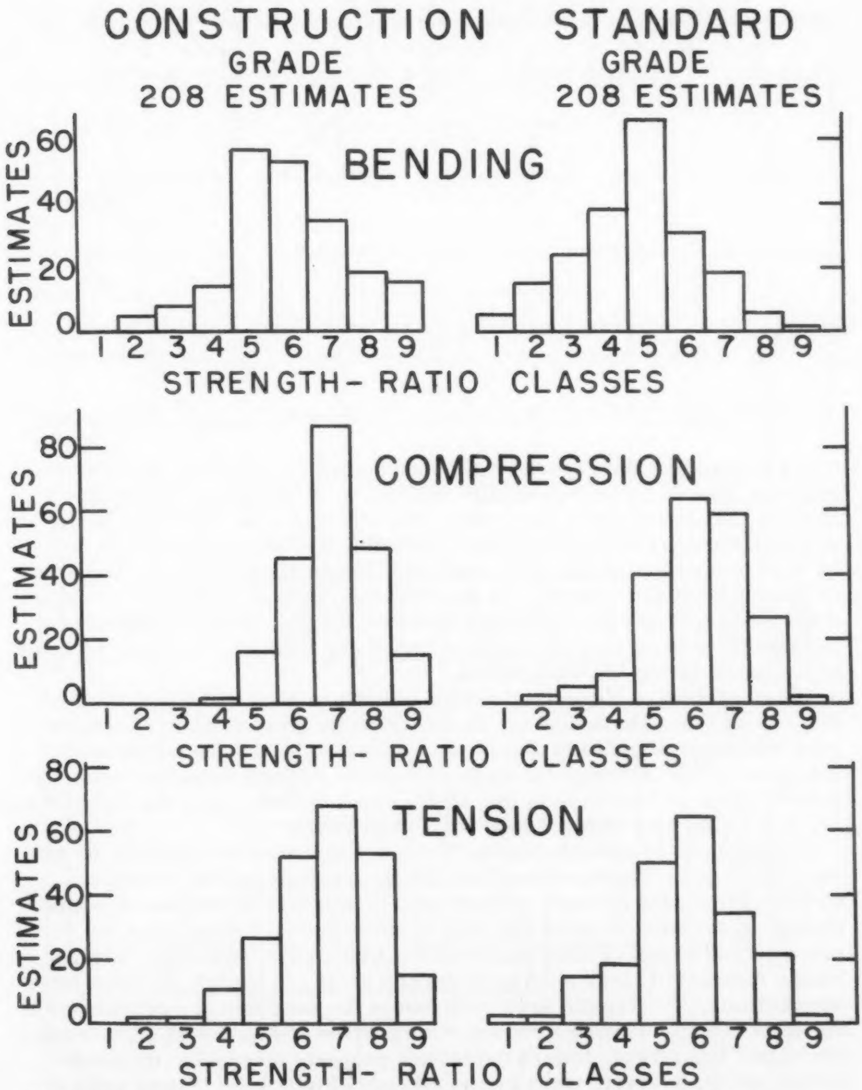

FIGURE I. DISTRIBUTION OF ESTIMATED STRENGTH RATIOS FOR CONSTRUCTION AND STANDARD GRADES OF 2 BY 4 INCH DOUGLAS FIR CONSTRUCTION LUMBER, IN CLASSES LISTED IN TABLE 2.

Table 3. Characteristics and Sources of Information for Strength of Clear, Unseasoned Coast and Interior West Douglas Fir as Determined from Standard Tests of Small, Clear, Specimens.

Property	No. values	Mean	Approximate CV*	Range	Source
		Psi	Per cent	Psi	
Bending	160	7,480	15	4,910-9,650	Forest Products Research Center
Compression	4,477	3,660	19	1,580-5920	U. S. Forest Products Lab.
Tension	257	12,280	25	2,700-24,000	Oregon State Col., Division Forest Research

* CV is coefficient of variation or standard deviation as per cent of mean.

The figures show that departure of actual values from normal distribution is generally typical in low probability regions, or in the "tails" of the distribution. This is not a new discovery. The effect of some skewness of the distributions, however, is to cause interesting distortions away from normality when they are combined with similarly skewed distributions by formation of random products. Because of this situation, it seemed desirable to manage clear-wood strength data as actual distributions, i.e. as individual values, and thereby develop product distributions giving theoretically close estimates of strengths among full-size pieces.

A comparison of clear-wood moduli of rupture used in the present study (Fig. 2) with the distribution for an outstandingly large number of rupture observations is afforded by Fig. 5 reproduced from U. S. Forest Service information.(13) Although the range in rupture strength detected during the present study is smaller than that shown by government data, the distributions are much alike over most of their coincident range.

Distributions of ultimate tensile strength for unseasoned Douglas fir are shown in Fig. 4. Tensile strength is the most variable of the principal strength properties of wood, as evidenced by standard deviations of tensile strength from about 24 to 29 per cent of mean values, respectively, for unseasoned and seasoned clear material.(8) Despite the variability, average tensile strength of clear wood is remarkably high, as limited previous tests have indicated.(6) Tensile working stresses for wood now are established at the same level as bending stresses. Comparison of Fig. 4 with Figs. 2 and 5 shows that this attitude toward the tension property may be too moderate. Unfortunately, however, there are no records of full-scale tension tests of structural lumber to support such assertion.

Product Frequency Distributions

Frequency distributions of random products expressing the short-time load-duration strength of unseasoned Douglas fir two- by four-inch lumber of Construction and Standard grades are shown in Figs. 6 to 10. A summary of

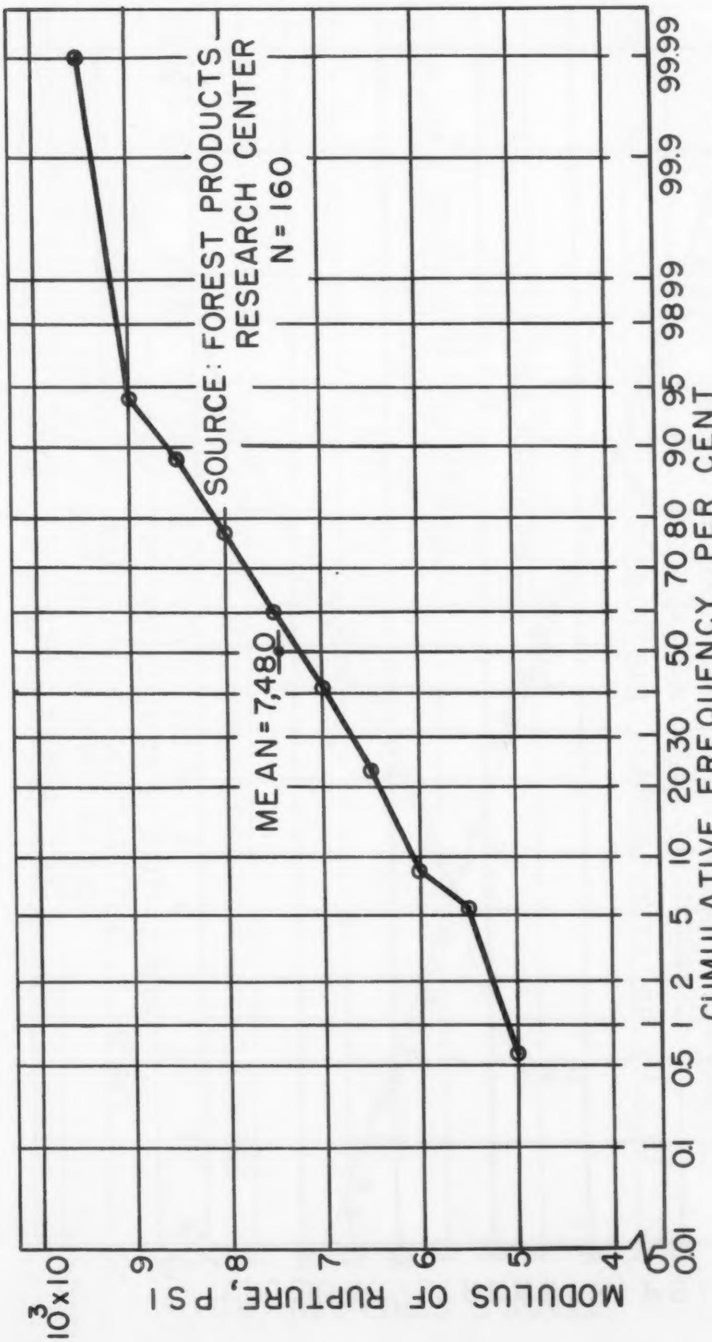

FIGURE 2. DISTRIBUTION OF MODULUS OF RUPTURE, CLEAR UNSEASONED DOUGLAS FIR.

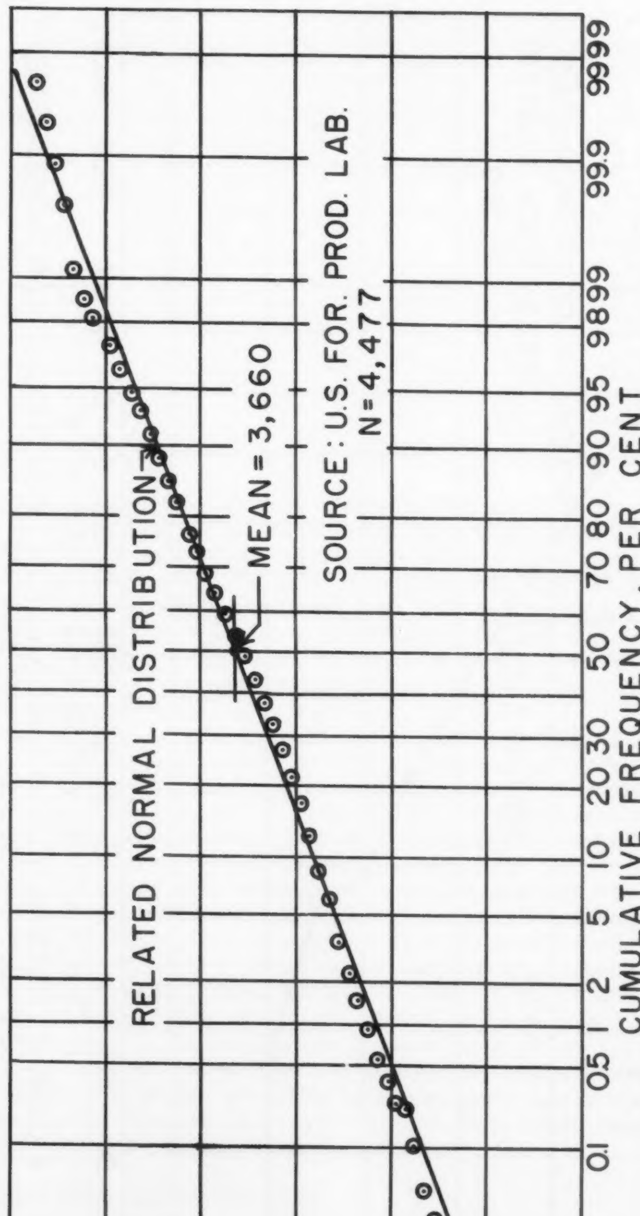

FIGURE 3. DISTRIBUTION OF MAXIMUM CRUSHING STRENGTH, CLEAR UNSEASONED DOUGLAS FIR.

U.S. FOREST SERVICE DATA.

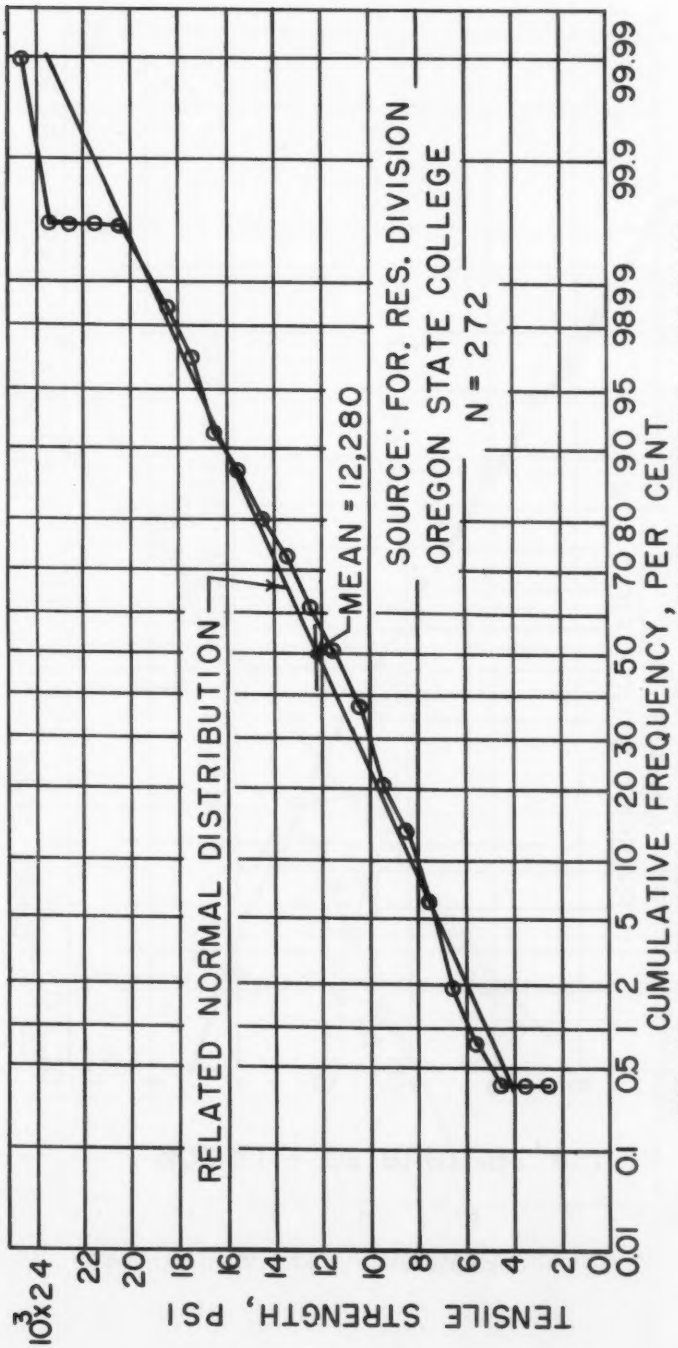

FIGURE 4. DISTRIBUTION OF ULTIMATE TENSILE STRENGTH, CLEAR UNSEASONED DOUGLAS FIR.

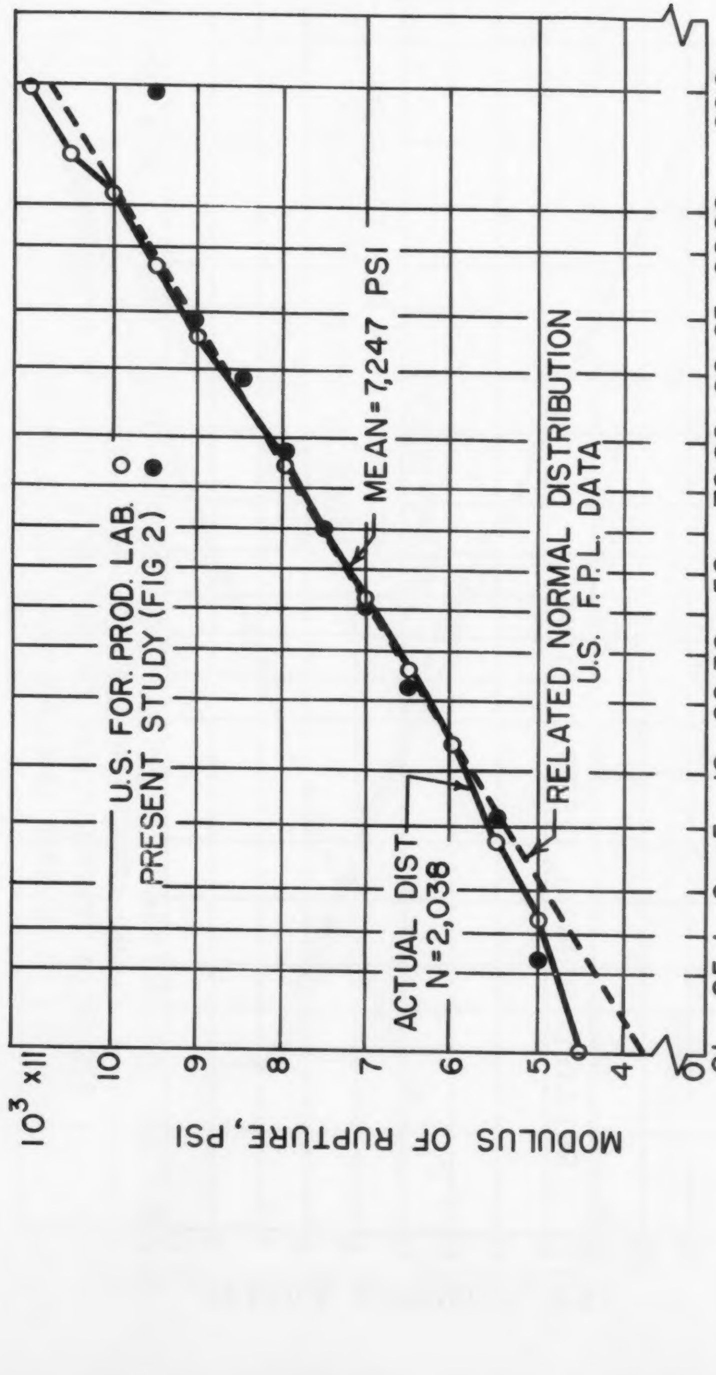

FIGURE 5. MODULUS OF RUPTURE, CLEAR UNSEASONED DOUGLAS FIR.

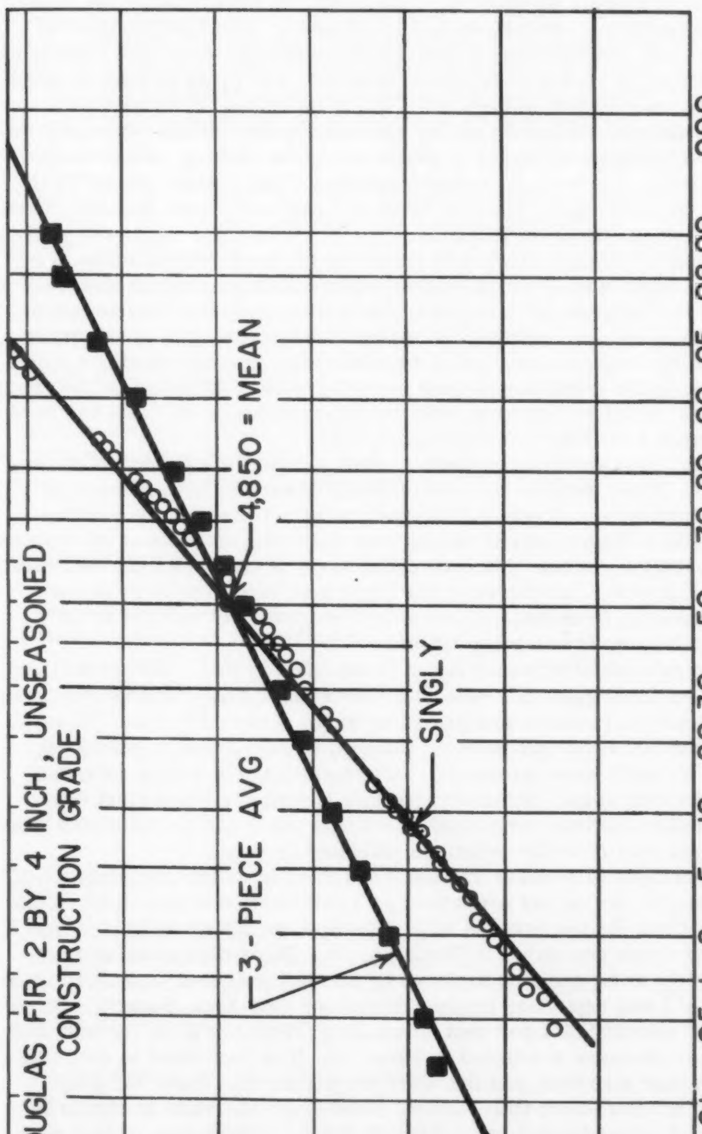

FIGURE 6. DISTRIBUTIONS OF BENDING STRENGTHS, SHORT-TIME LOAD CONDITIONS.

some pertinent statistics of these distributions is given in Table 4. Complete distributions of bending strength for each grade, as generated by the random products computations, are shown in Figs. 6 and 7. Enlarged portions of these and similar distributions to provide a magnified view of the vicinity of low probability, i.e., a low cumulative frequency, are given in Figs. 8 (bending), 9 (compression) and 10 (tension). Attention should be focused at or about 1 per cent cumulative frequency for consideration of safe stress levels.

Presumed breaking strengths of single pieces in bending, compression, and tension under short-time loading conditions ("short-time stress") are indicated at the arbitrarily selected level of 1 per cent in the figures. These stresses are summarized in Table 5A.

Comparison of straight lines with irregular ones, or with patterns of plotted points, in Figs. 6 to 10 revealed that intersections of product distribution curves with the ordinate for 1 per cent cumulative frequency varied considerably. Straight lines are segments of normal distribution curves related to connected points representing actual distributions. The intersections with 1 per cent cumulative frequency of each curve have been darkened on the charts for emphasis. In a few instances, only the intersection point itself has been placed, to avoid a confusion of lines on the charts.

It is evident that product distribution curves depart for normality in the "tail" region. Their general trend is to swing upward in such manner as to cause the intersections of actual distributions with 1 per cent cumulative frequency to be at higher stress values than such intersections of the related normal distribution curves. Because differences in stresses selected from actual and related normal curves at the 1 per cent level are large in many instances, stresses from both curves have been specially marked in the figures and tabulated in summary tables.

The interpretation of stresses listed in the table is that a failure will occur in 1 of 100 times randomly selected, unseasoned, single pieces of Douglas fir two- by four-inch lumber are loaded up to the stress indicated. Theoretically, 99 of the 100 randomly selected pieces will have little to extensive reserves of strength when stressed to only slightly above values selected from the 1 per cent level. Stresses pertinent to longer odds against failure (under short-time loading conditions) of a single piece can be estimated from curves in Figs. 6 to 10 to the extremes validated by data.

Average strength of 3 values randomly selected from the distributions of strength of single pieces was expressed as a distribution of averages. Complete distributions for the "groups of 3" situation are shown in Figs. 6 and 7, and magnified views are shown in Figs. 8 to 10. These distributions were derived from those for individual pieces by dividing selected deviations from the mean by $\sqrt{3}$ and replotting the new deviations (see Figs. 6 and 7). Stresses were then selected at 1 per cent cumulative frequency from the adjusted curves. These stresses are listed in Table 5B. It is important to note that these are average stresses, and that they are higher than those for single pieces. They are for short-time loading conditions. Increase in stress for the groups-of-3 situation occurs because of chance combination of high with low individual values. Adjusted distributions depicting this situation are tipped more toward a horizontal position than are distributions representing strengths of single pieces. Stresses indicated in Table 5B can be interpreted to mean that a group of 3 members will bear an average stress 1 in 100 times that would rupture similarly stressed, single pieces more frequently than 1 in 100 times.

Table 4. Statistical Characteristics of Product Frequency Distributions for Bending, Crushing and Tensile Strength of Construction and Standard Grades Douglas Fir 2- by 4-inch Dimension Lumber.

Item	Construction grade	Standard grade
	Psi	Psi
Bending Strength		
Mean computed for product distribution	4,846	3,798
Standard deviation of product distribution	1,406	1,359
2.326 X Standard deviation	3,270	3,161
$2.326 \times \left[\frac{\text{Standard deviation}}{\sqrt{3}}\right]$	1,889	1,826
Median estimated from actual distribution	4,670	3,700
Deviation of actual value at 1% of products from median	2,920	2,800
Crushing Strength		
Mean computed for product distribution	2,709	2,366
Standard deviation of product distribution	646	623
2.326 X Standard deviation	1,503	1,449
$2.326 \times \left[\frac{\text{Standard deviation}}{\sqrt{3}}\right]$	868	837
Median estimated from actual distribution	2,650	2,366
Deviation of actual value at 1% of products from median	1,190	1,406
Tensile Strength		
Mean computed for product distribution	8,265	6,925
Standard deviation of product distribution	2,740	2,560
2.326 X Standard deviation	6,373	5,955
$2.326 \times \left[\frac{\text{Standard deviation}}{\sqrt{3}}\right]$		
Median estimated from actual distribution	8,300	7,100
Deviation of actual value at 1% of products from median	4,900	5,200

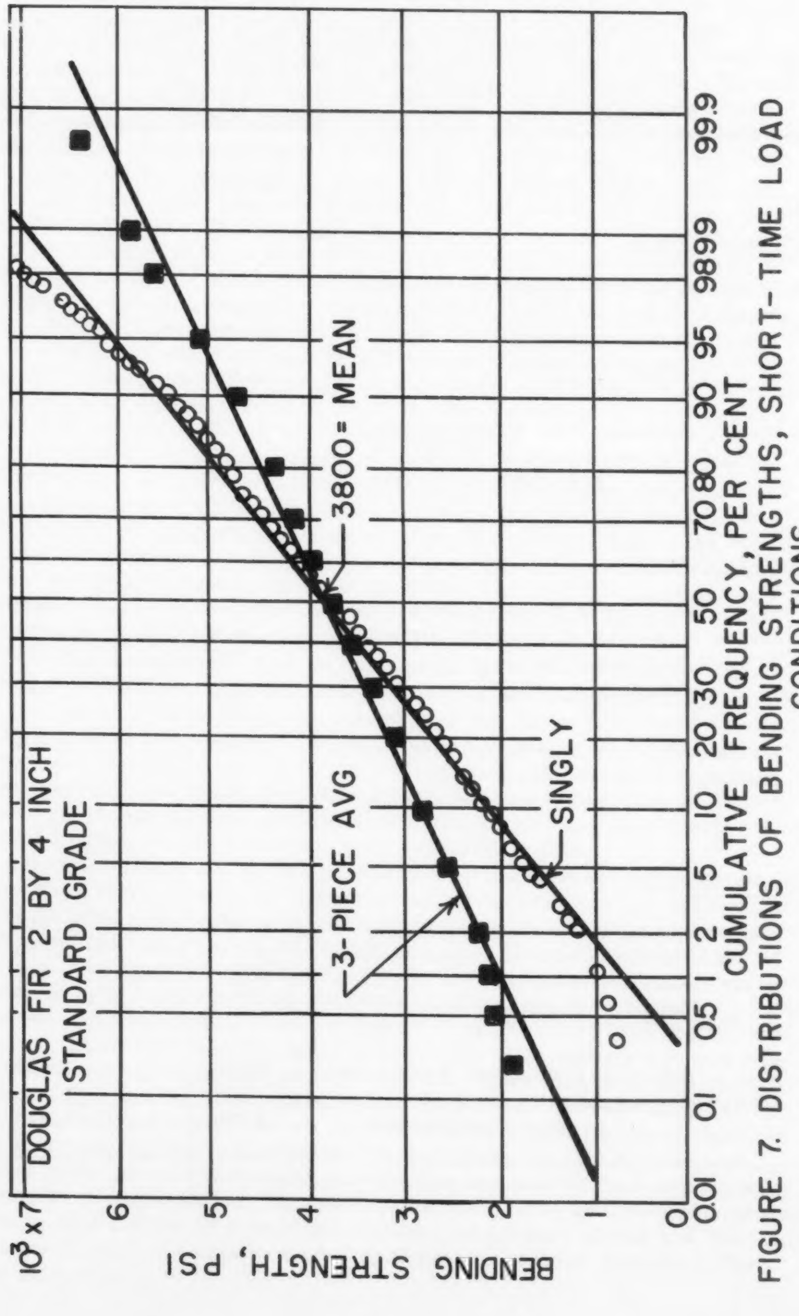

FIGURE 7. DISTRIBUTIONS OF BENDING STRENGTHS, SHORT-TIME LOAD CONDITIONS

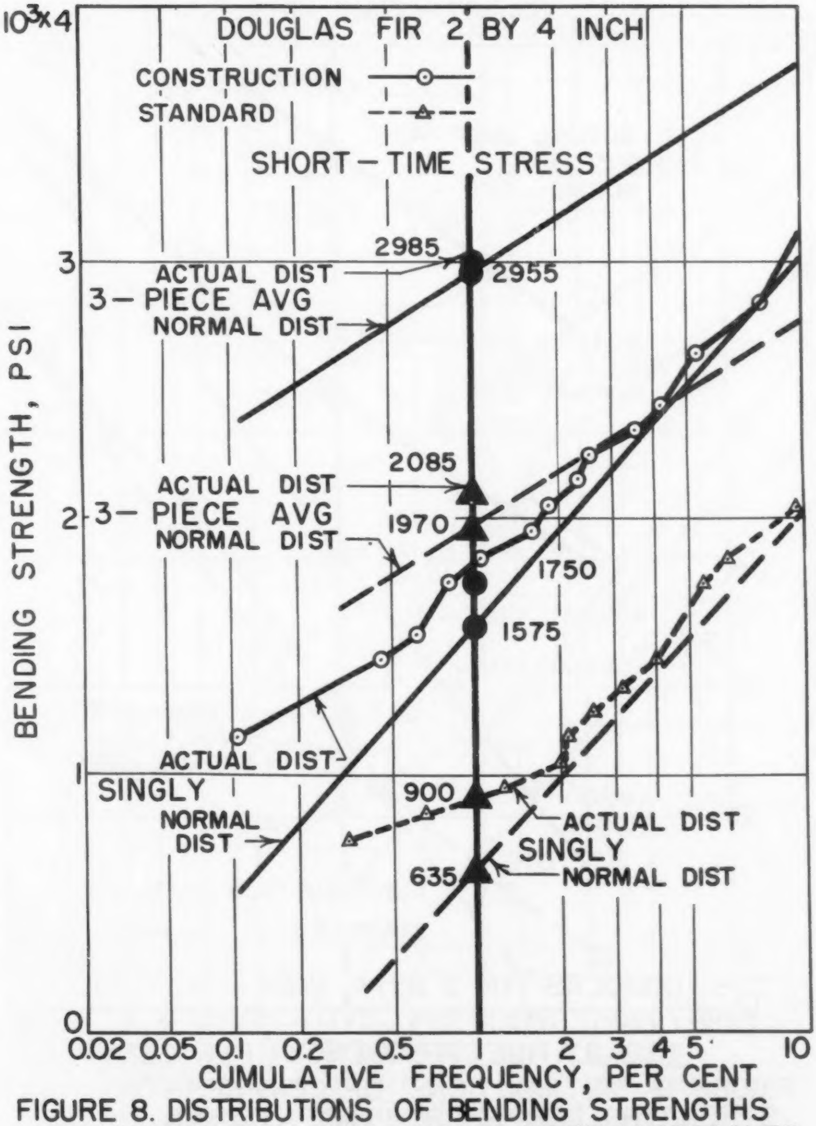

FIGURE 8. DISTRIBUTIONS OF BENDING STRENGTHS FOR SHORT-TIME LOADING, LUMBER UNSEASONED.

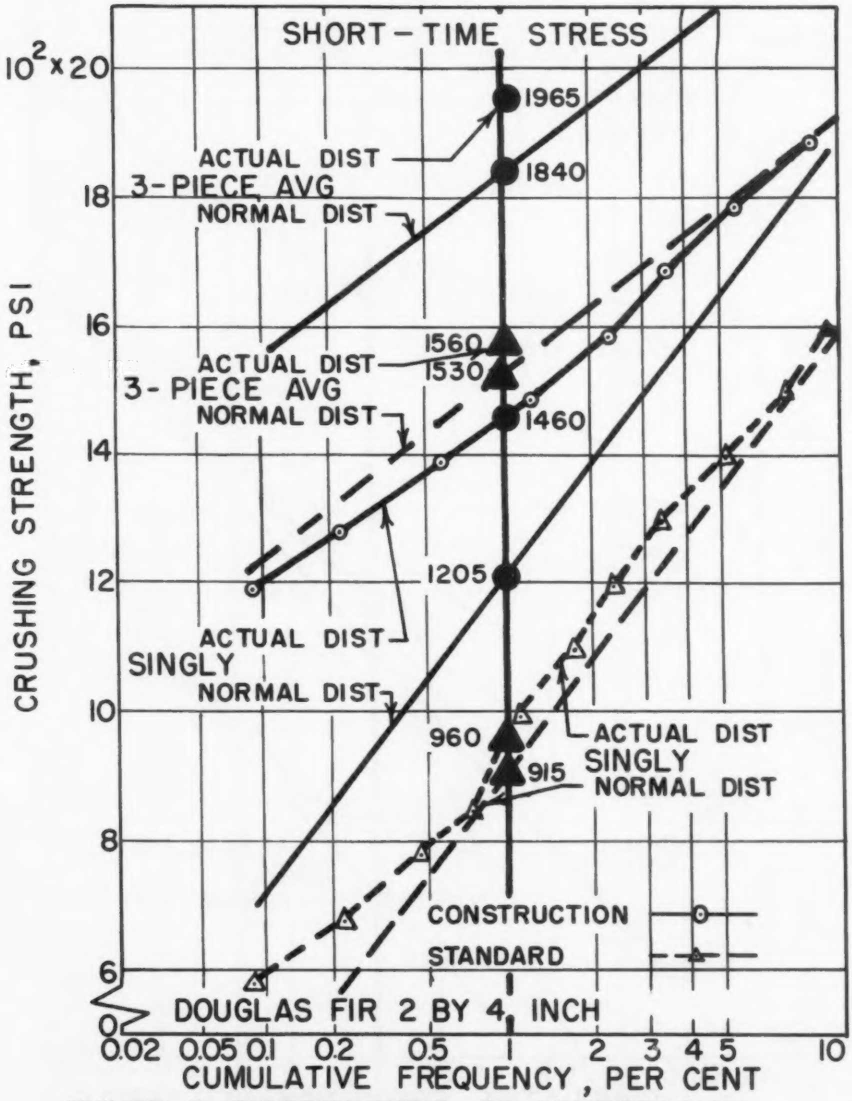

FIGURE 9. DISTRIBUTIONS OF COMPRESSION STRENGTHS FOR SHORT-TIME LOADING, LUMBER UNSEASONED.

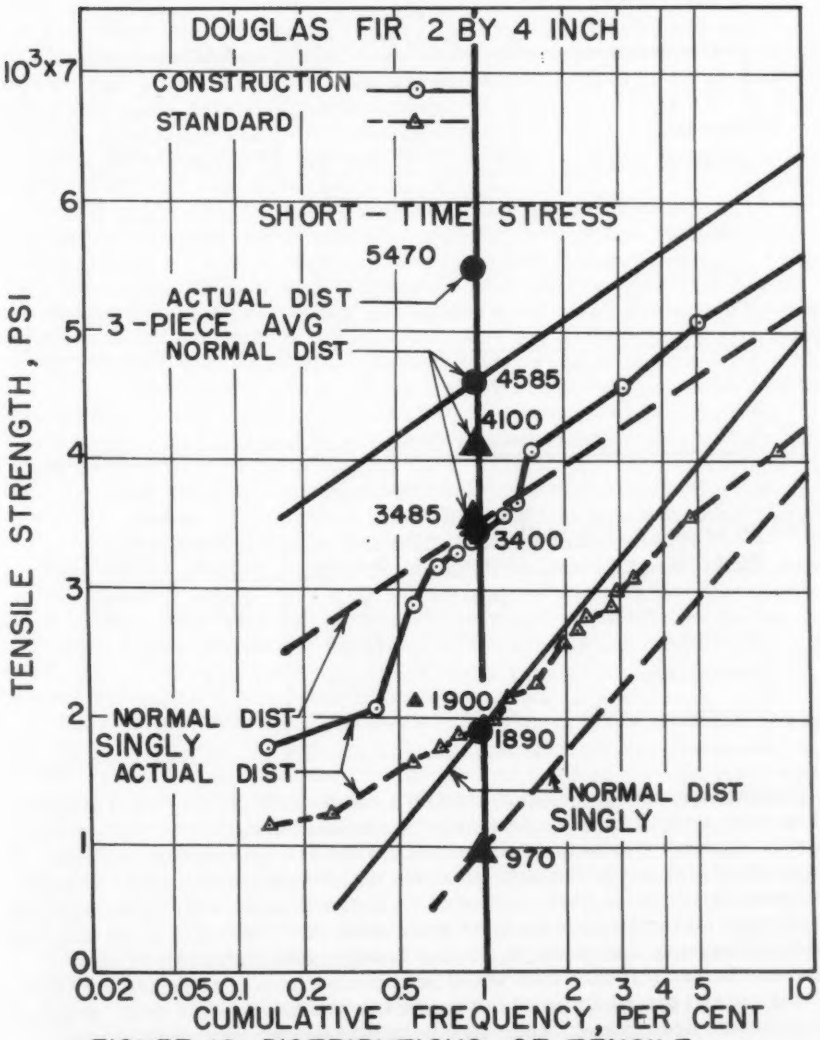

FIGURE 10. DISTRIBUTIONS OF TENSILE STRENGTHS FOR SHORT-TIME LOADING, LUMBER UNSEASONED.

Table 5. Stresses for Short-Time Load-Duration Conditions in Bending, Compression and Tension for Construction and Standard Douglas Fir 2- by 4-inch Dimension Lumber, Unseasoned.

A. Pieces Used Singly.

Strength property	Construction grade Distribution		Standard grade Distribution	
	Actual	Normal	Actual	Normal
	Psi	Psi	Psi	Psi
Bending	1,750	1,575	900	635
Compression	1,460	1,205	960	915
Tension	3,400	1,890	1,900	970

B. Pieces Used in Groups of Three (Average Stress).

Strength property	Construction grade Distribution		Standard grade Distribution	
	Actual	Normal	Actual	Normal
	Psi	Psi	Psi	Psi
Bending	2,985	2,955	2,085	1,970
Compression	1,965	1,840	1,560	1,530
Tension	5,470	4,585	4,100	3,485

Comparisons are provided by Tables 4 and 5 and Figs. 6 to 10 of stresses taken from actual and related normal distribution curves. It is evident that relationships between actual and normal curves are not constant, but that actual distributions consistently lie above the related normal ones. This performance is a function of the nature of the original data on strength-ratio and clear-wood strength and phenomena associated with combining these data into product frequency distributions. In any event, actual distributions show generally higher stresses than would be expected from considering only the related normal distributions of the respective random product data. Some of the contrasts are striking, whereas others may be only chance variations. Except for crushing strength, stresses selected at the 1 per cent level from actual distributions for Standard grade (Table 5A) are pronouncedly higher than those selected from the related normal distributions; differences expressed as a per cent of the stress from normal curves range from 96 for tension to 5 for compression. For the Construction grade (Table 5A) the range is 80 for tension to 11 for bending. Similar relationships between stresses from actual and related normal distributions for 3-piece averages (Table 5B) show closer agreement. Greatest differences are less than 20 per cent of the value from normal distributions.

Influence of Seasoning

The distribution of relative strength increases occurring when clear, unseasoned wood dries is shown by the array of increase values listed in Table 6. When this distribution was combined with bending SR and clear-wood moduli of rupture in a three-way random product analysis, the resulting distributions (for single pieces only) are as shown in Fig. 11. To provide comparisons, the same distributions have been redrawn in Figs. 12 and 13, overlaid on the distributions for bending strength of full-size, unseasoned, material. Stresses computed or selected from the 1 per cent cumulative frequency level of Fig. 11 are listed in Table 7.

Curves in Figs. 11, 12, and 13 describing strength distributions of seasoned Construction and Standard Douglas fir two- by four-inch lumber, are useful only to indicate maximum increases theoretically possible. Actual increases realized will be considerably lower, because evidently the full potential increases do not occur. Limited evidence of this situation is provided by ragged distributions of strength values of dry 2- by 4-inch pieces tested during the check-test phase of the present study as shown in Figs. 12 and 13, Curve (C). These distributions fail, by a considerable margin, to correspond with curves showing the theoretically attainable strength of dry material. The number of pieces tested to provide this evidence apparently was insufficient (only about 80) to define smooth distribution curves for strength of dry material. In the middle and at the right ends, theoretical and actual distribution curves show some signs of parallelity, and test pieces from both grades exhibited strengths superior to those theoretically characteristic of unseasoned stock. However, to the left of the mean (below 50 per cent of cumulative frequency) charted data from the actual tests indicate that Standard grade pieces (tested dry) did not measure up to levels unseasoned pieces theoretically should achieve. Construction grade material, based on the tests of dry stock, followed closely the distribution presumably characteristic of unseasoned stock for stress levels below the mean.

It has long been noted that dimension lumber from 2 to 4 inches thick and having low strength-ratio benefits little or none from seasoning; the coarse defects present, when also seasoned, depreciate gains in strength made when clear wood in such lumber seasons. This performance is not typical of material of high strength ratio, where apparently net gains in strength due to seasoning generally occur. Results pictured in Figs. 12 and 13 seem to confirm that notion. It is possible that strength gains in dimension lumber due to seasoning justify somewhat more allowance when working stresses are set than now permitted. But even so, it is evident that optimum gains estimated in the present study probably cannot be achieved by lumber with any level of strength ratio.

The mean modulus of rupture for short-time loading of clear Douglas fir (species average based on tests of small specimens) recently was reported as 7,460 psi.[4] The mean bending strengths of Construction and Standard unseasoned two- by four-inch lumber taken from product distributions are about 4,850 and 3,800 psi, respectively (Figs. 6 and 7). Tests of dry lumber (Figs. 12 and 13) indicated that some improvement in bending strength begins to appear at stress levels of about 3,500 psi in Construction grade and 4,000 psi in Standard grade. Associated probabilities (cumulative per cent frequency) for these stress levels are about 14 and 50 per cent, respectively, for the grades. If the probability scale is transposed to one of "equivalent strength

Table 6. Per Cent Increases in Modulus of Rupture of Clear, Unseasoned Douglas Fir Due to Drying to about 12 Per Cent Moisture Content. Standard Tests of 129 Pairs of 1- by 1-inch Specimens from 8- and 12-foot Douglas Fir 2- by 4-inch Dimension Lumber Randomly Selected at 11 Sawmills.

Strength increase classes	Obs. per Class	Individual values of $\left(\frac{\text{dry strength}}{\text{green strength}} - 1\right)100$, per cent												
10.0-19.9	2	11.4	19.7											
20.0-29.9	4	23.4	24.4	28.1	29.7									
30.0-39.9	2	30.2	33.3											
40.0-49.9	10	40.0	40.2	40.8	41.5	43.6	44.1	45.5	47.0	48.7	49.6			
50.0-59.9	13	51.5	51.7	52.7	52.9	53.5	54.5	55.0	56.9	57.0	57.9	58.4	58.5	59.9
60.0-69.9	19	60.0	60.2	60.6	61.6	61.7	62.3	63.7	64.0	64.0	64.1	65.0		
		65.9	66.0	66.2	68.0	68.5	68.9	69.4	69.8					
70.0-79.9	26	70.2	70.3	70.5	71.7	71.9	73.0	73.5	73.8	73.8	73.9	74.0	75.1	75.8
		75.9	76.1	76.2	76.2	76.4	76.4	76.7	77.4	77.9	78.8	78.9	79.0	79.0
80.0-89.9	23	80.3	81.3	81.4	81.6	82.0	82.6	83.0	83.9	84.1	84.4	84.4	85.0	
		85.0	85.2	85.6	85.8	86.7	87.0	88.9	88.9	89.1	89.3	89.4		
90.0-99.9	16	90.5	91.1	91.2	92.0	92.4	93.4	94.1	94.7	94.9	95.0	95.6		
		95.6	95.7	96.8	98.9	99.6								
100.0-109.9	11	100.3	101.0	101.7	101.8	102.0	102.1	103.3	105.0	105.2	107.0	108.2		
110.0-119.9	3	112.0	115.0	118.0										
TOTAL	129	AVERAGE 73.8 PER CENT												

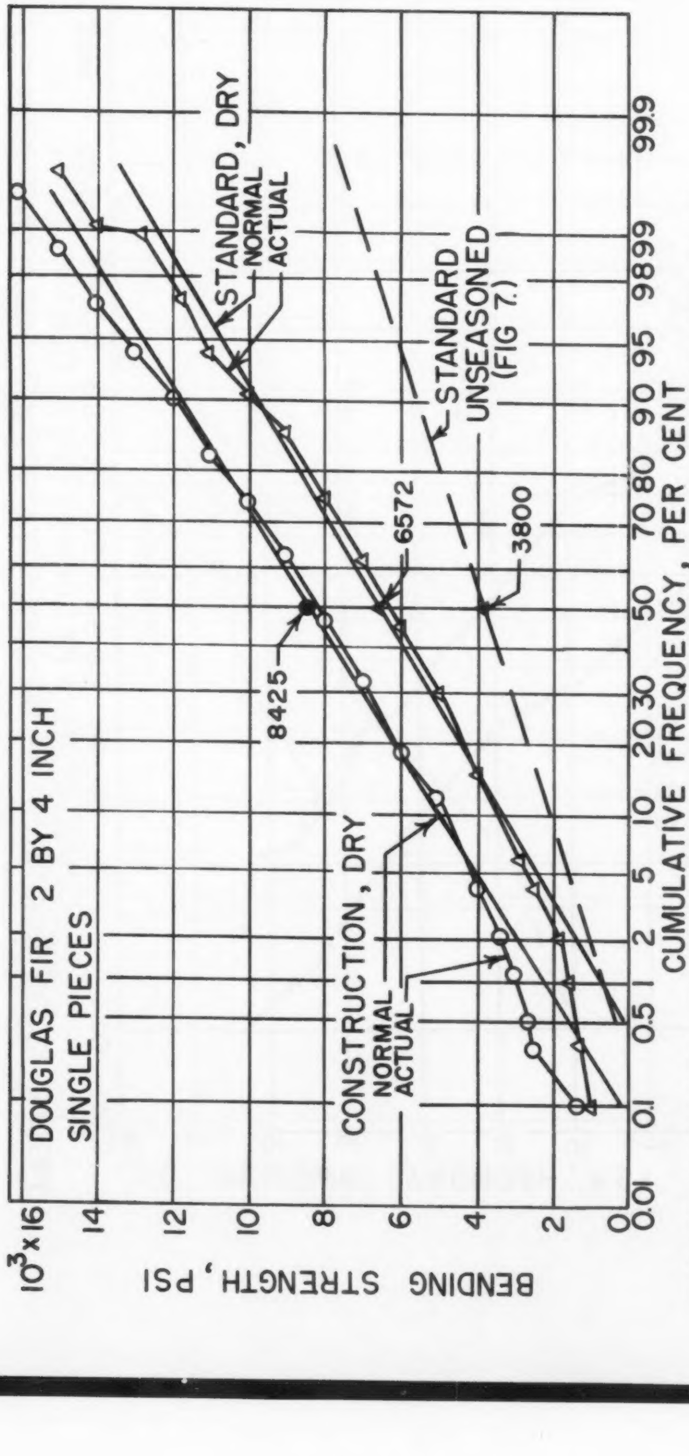

FIGURE 11. DISTRIBUTIONS OF BENDING STRENGTHS, IF PIECES DEVELOP THEORETICAL MAXIMUM STRENGTH AFTER SEASONING.

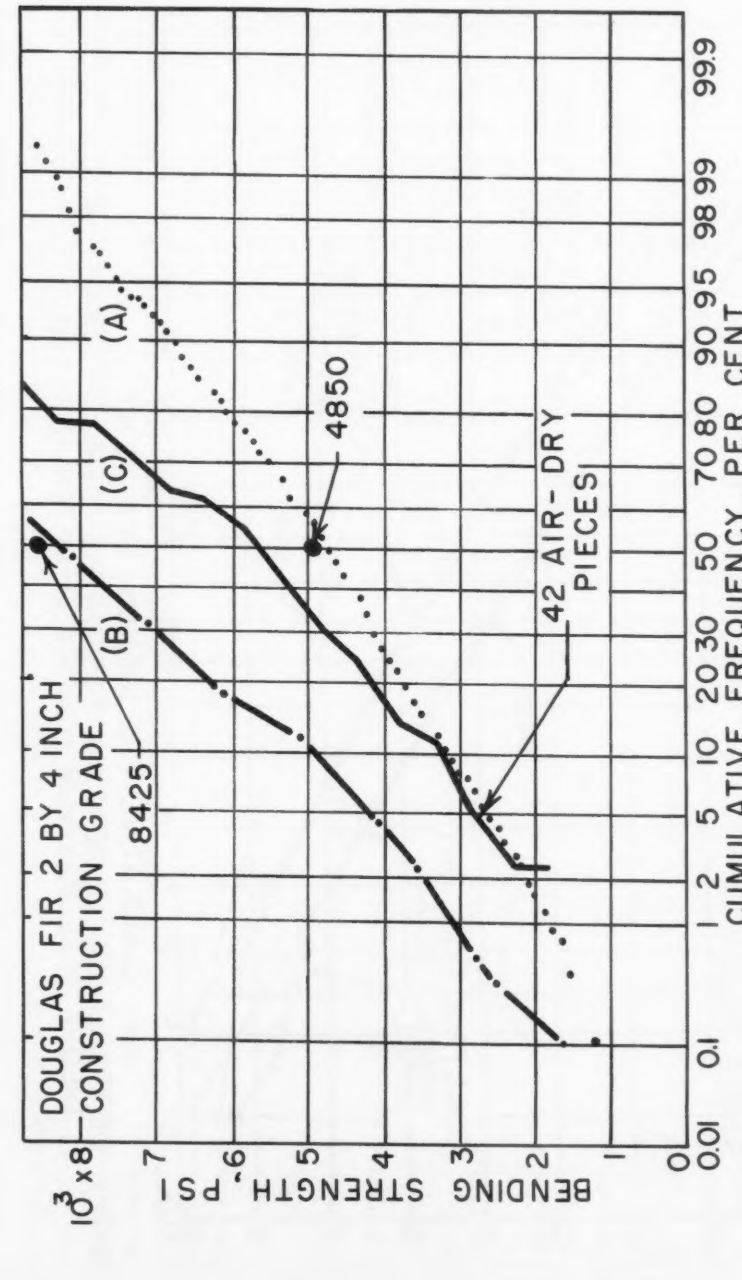

FIGURE 12. DISTRIBUTIONS OF BENDING STRENGTH, (A) UNSEASONED (FIG. 6), (B) SEASONED (FIG. 11), (C) SEASONED (TEST DATA).

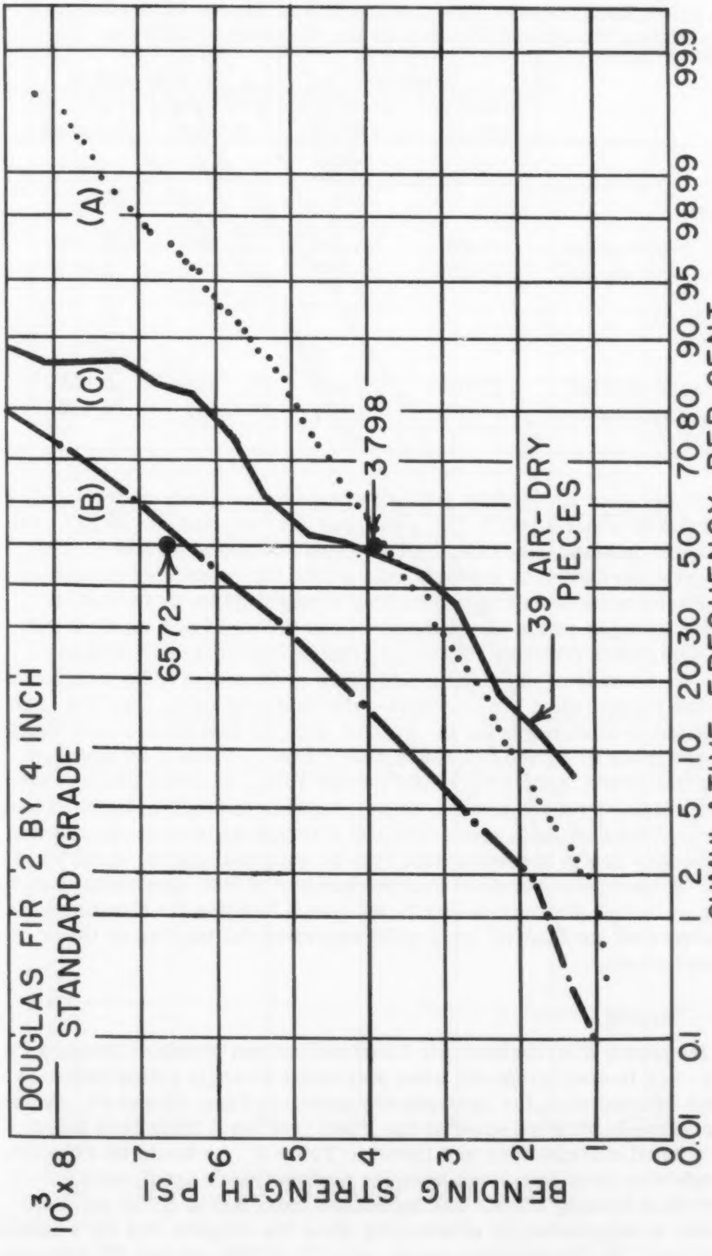

FIGURE 13. DISTRIBUTIONS OF BENDING STRENGTH, (A) UNSEASONED (FIG. 7), (B) SEASONED (FIG. 11), (C) SEASONED (TEST DATA).

Table 7. Stresses under Short-Time Loading Conditions for Construction and Standard Douglas Fir 2- by 4-inch Dimension Lumber: Maximum Stresses Theoretically Possible for 99% of Pieces Perfectly Seasoned, Compared with Stress for Unseasoned Material.

Grade	Seasoned		Unseasoned	
	Individual pieces	Groups of 3 pieces	Individual pieces	Groups of 3 pieces
	Psi	Psi	Psi	Psi
Construction				
Actual Distribution	3,050	5,225	1,750	2,985
Normal Distribution	2,150	4,800	1,575	2,955
Standard				
Actual Distribution	1,700	3,615	900	2,085
Normal Distribution	760	3,215	635	1,970

ratio", the 14 per cent probability level in Construction grade corresponds to a strength ratio of about 3,500/7,460, or 47 per cent SR, and the 50 per cent probability level in Standard grade to about 4,000/7,460, or 54 per cent SR. The important contribution to strength gain within the respective grades apparently came from the small to pronounced strength gains by individual pieces having strength ratios above those values (47 or 54 per cent), while individuals with lower strength ratios contributed nothing to strength improvement. In the case of Standard grade there even seems to be a depreciating influence caused when low-strength-ratio material dries (see Fig. 13).

The mean gains in strength for the grades, with all individuals contributing their shares, appear to be 100 [(6,265/4,850) - 1.00], or about 29 per cent for Construction grade, and 100 [(4,705/3,800)- 1.00], or about 24 per cent for Standard grade. Present results, therefore, tend to confirm the idea that low SR material does not gain appreciably in strength as it seasons, and that average increases due to seasoning this type of material may be about 24 to 29 per cent. It must be recognized that the number of test specimens used to provide data on actual strength of dry 2- by 4-inch Douglas fir dimension lumber is somewhat too limited for a solid experimental backing of these tentative conclusions.

Influence of Changing Grade

Product frequency distributions for Construction and Standard Douglas fir two- by four-inch lumber prepared when estimated strength ratios below 38 per cent were ejected from the analysis are shown in Figs. 14 and 15. Stresses for short-time loading selected at the 1 per cent level from both actual and related normal distributions are listed in Table 8. As would be expected, Standard grade was most improved when its contents were artificially culled.

The short-time loading stress was increased from 900 to 2,100 psi, and this result was accomplished by eliminating 44 of the original 208 SR's falling below 38 per cent. In Construction grade, only 13 of 208 original SR estimates

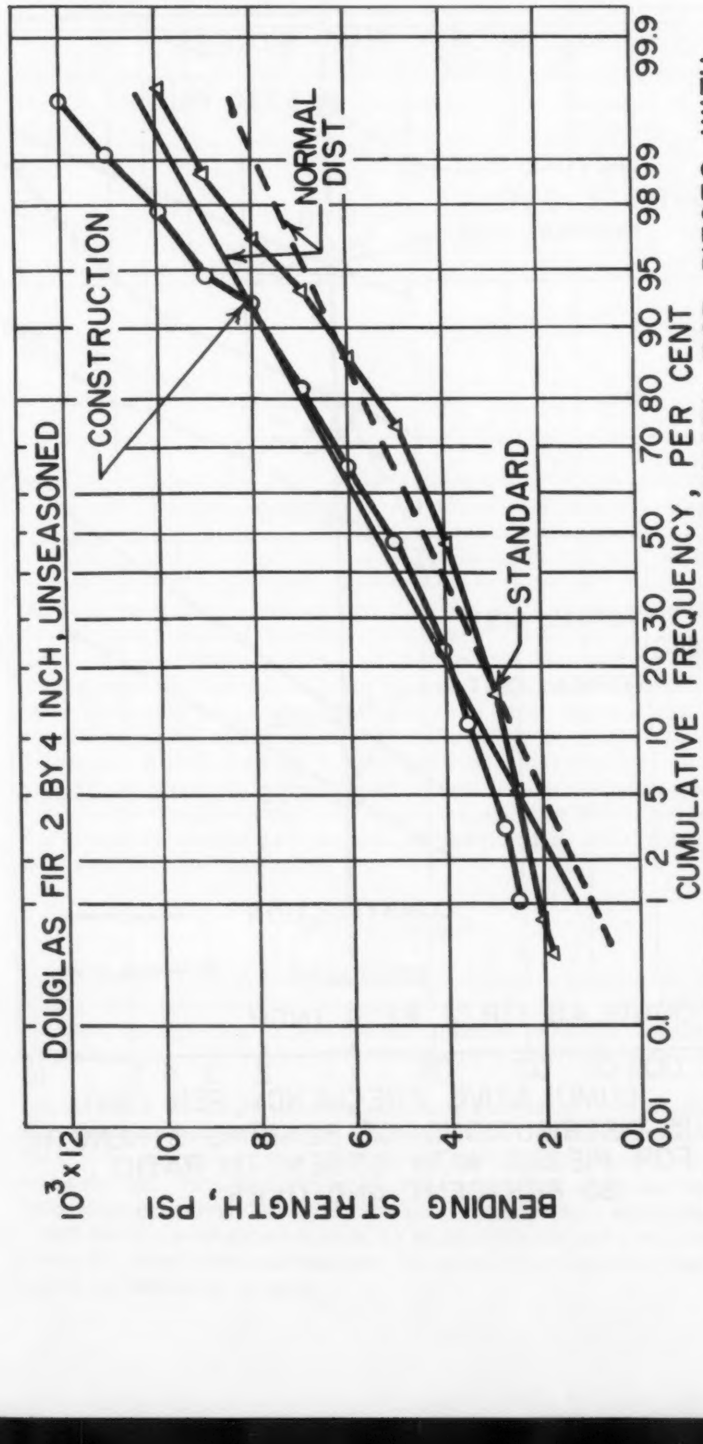

FIGURE 14. DISTRIBUTIONS OF BENDING STRENGTH FOR PIECES WITH STRENGTH RATIO 38 PER CENT AND OVER.

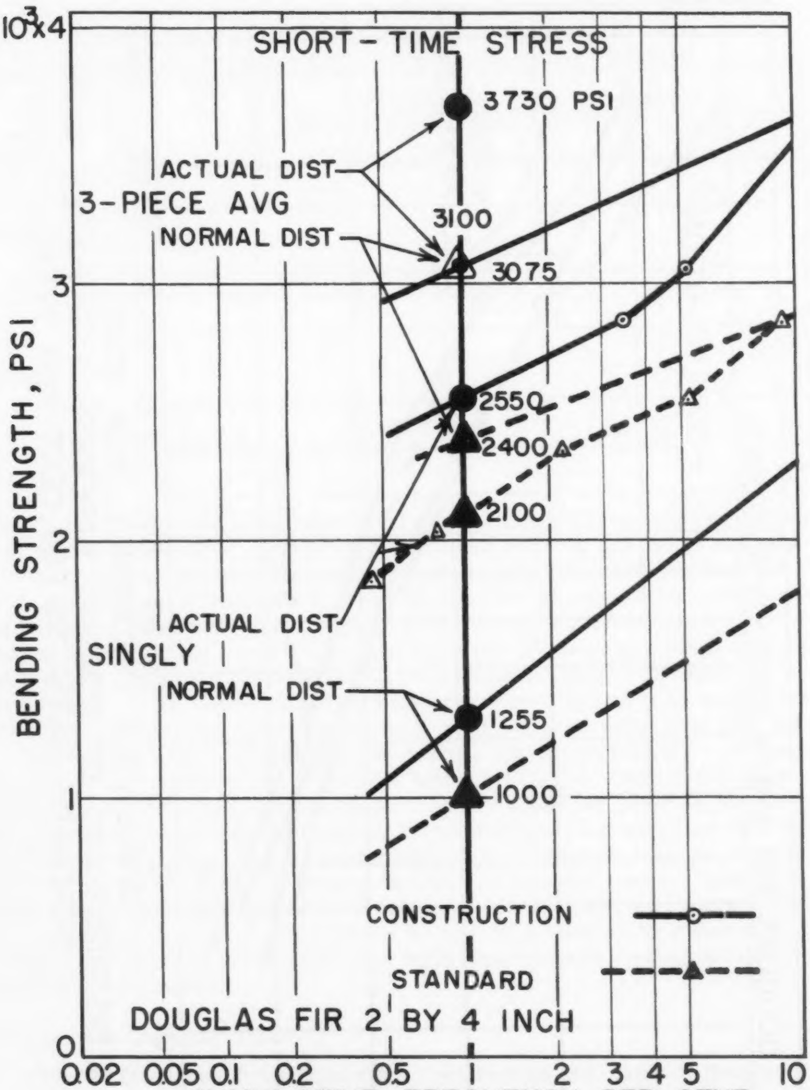

FIGURE 15. DISTRIBUTIONS OF BENDING STRENGTHS FOR PIECES WITH STRENGTH RATIO 38 PER CENT AND OVER.

Table 8. Stresses Selected From 1% Distribution for Short-Time Loading Conditions of Construction and Standard Douglas Fir 2- by 4- inch Dimension Lumber When Strength Quality Range of Grades was Altered.

Grade	Individual pieces	Averages of 3 pieces
	Psi	Psi
Construction		
Actual Distribution	2,550	3,730
Normal Distribution	1,255	3,075
Standard		
Actual Distribution	2,100	3,100
Normal Distribution	1,000	2,400

fell below 38 per cent, but elimination of these fringe values caused the random product distribution to elevate bending stress by about 800 psi at the 1 per cent level.

Interpretation of the above results should be made with full recognition that distributions of computation input data were especially modified for this phase of the experiment. The modification was about comparable to reducing the size of maximum knot permitted in Standard grade from 2- to 1-1/2-inch diameter. Coupled with adjustment of data was the assumption that all eliminated SR's were low because of large knot size in lumber originally inspected, but were not low for some other reason such as steep slope of grain.

It is not suggested that present grades of lumber be changed in view of these results. It is interesting to note, however, that distributions of estimated strength, as prepared in the present project, are sensitive to manipulation. This observation suggests that lumber grades might be developed to exacting, special stress requirements if demand for them arose, and if costs of selecting such grades were reasonable.

Working Stresses

The objective of the present study was to develop a possible basis for determining reasonable working stresses for two- by four-inch lumber in 2 typical nonstress grades. It was not the objective to derive, or suggest, such working stresses. A few general comments on this subject, however, will be made.

The stresses for short-time load duration estimated from product frequency distributions must be adjusted downward before values of practical use in design can be expressed. One common and well-known adjustment is for duration of load. This factor is applied in recognition of the fact that strength of wood is progressively higher as the load-duration time diminishes. For practical design, load duration must be reckoned with, and a reduction of stresses for short-time load duration (as revealed by standard tests, for example) is, therefore, proper.

Reductions are necessary to take care of other aspects of design problems or special characteristics of material generally thought to influence its performance in structural applications. Considerable study has been made of factors influencing the strength of wood as such and as used in practical design.(14,15) More study of these matters perhaps will be made, following lines of thought already proved fruitful. The method of establishing a basis for determining working stresses in wood used during the present project may have merit, if applied to further extend knowledge of the properties and performance of timber of greatest interest to the engineer.

There is no suggestion from present work that working stresses for wood should be altered drastically. Traditional practices of stress grading and establishing working stresses for individual timbers in the conventional stress grades have been satisfactory in the past. It is possible, however, that improved information on the ranges of strength to be expected in both stress and nonstress grades could be gathered through additional studies of the sort described here. This type of information could be useful for application to design problems where multiple action of load-carrying members is known to take place.

Because of natural variation in the strength of both clear wood and commercial timbers, the vast majority of pieces in any grade seem to have strength more than adequate at design stresses now used. If, somehow, individual pieces of any given grade possessing superior strength properties could be separated from others in the grade, higher working stresses than now practical for wood design might be justified. Research leading to simple means of detecting superior material could pay large dividends to both structural engineer and lumber industry.

REFERENCES

1. American Society for Testing Materials. Tentative Methods for Establishing Structural Grades of Lumber. ASTM Designation D245-49T, 1949, and D245-57T, 1957.

2. -------------- Standard Methods for Testing Small Clear Specimens of Timber. ASTM Designation D143-52, 1952.

3. -------------- Standard Methods of Static Tests of Timbers in Structural Sizes. ASTM Designation D198-27, 1927.

4. Drow, John T. Relationship of Locality and Rate of Growth to Density and Strength of Douglas-fir. U. S. Forest Products Laboratory Report No. 2078, May 1957.

5. Freas, A. D. and Selbo, M. L. Fabrication and Design of Glued Laminated Wood Structural Members. U. S. Department of Agriculture Technical Bulletin No. 1069, February 1954.

6. Markwardt, L. J., and Wilson, T. R. C. Strength and Related Properties of Woods Grown in the United States. U. S. Department of Agriculture Technical Bulletin 479, 1935.

7. U. S. Department of Agriculture. Wood Handbook. Department Handbook No. 72, 1955.

8. Van Vliet, A. C. The Strength of Second-Growth Douglas-Fir in Tension Parallel to Grain. Oregon State College thesis, 1958.

9. West Coast Lumbermen's Association. Standard Grading and Dressing Rules No. 15, 1956.

10. Wilson, T. R. C. Guide to the Grading of Structural Timbers and the Determination of Working Stresses. U. S. Department of Agriculture Miscellaneous Publication No. 185, 1934.

11. Supplements 1 and 2, respectively, to Reference (10). U. S. Forest Products Laboratory Reports No. R1225, 1940 and R 1715, 1948.

12. Wood, L. W. Relation of Strength of Wood to Duration of Load. U. S. Forest Products Laboratory Report No. 1916, 1951.

13. Wood, L. W. Strength Qualities of Dimension Lumber. ASTM Paper No. 91 Second Pacific Area National Meeting, September 17-21, 1956.

14. Wood, L. W. Variation of Strength Properties in Woods Used for Structural Purposes. U. S. Forest Products Laboratory Report No. R1780, 1950.

15. Wood, L. W. Evaluation of the Factor of Safety in Structural Timbers. U. S. Forest Products Laboratory Report No. 2068, 1957.

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ANALYSIS OF A TWO WAY TRUSS SYSTEM

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SYNOPSIS

The analysis of a two way truss system forming a space grid is described. This grid can be rigidly or elastically supported as desired. Two methods of analysis are presented of which the first is based on the method of consistent deflections and the second on the solution of an anisotropic flexural grid. Application of the methods in the analysis and design of the U. S. Air Force Academy Dining Hall at Colorado Springs, Colorado is discussed.

INTRODUCTION

The architectural and functional requirements for the Dining Hall of the United States Air Force Academy at Colorado Springs, Colorado called for a relatively thin flat roof structure which would provide a clearspan interior area of 266' x 266' with a 21' cantilever around the perimeter. In order to meet these requirements it was decided to use an orthogonal two way truss system forming a space grid supported on sixteen columns. Analysis of this type of roof by two methods is presented. The first analysis is based on the method of consistent deflections. For the second analysis the roof is considered as an equivalent anisotropic flexural grid and treated by the method of finite differences.

Description of Structure

A plan view and elevation of the roof structure considered is shown in Fig. 1. The roof structure is square in plan measuring 308' x 308' between

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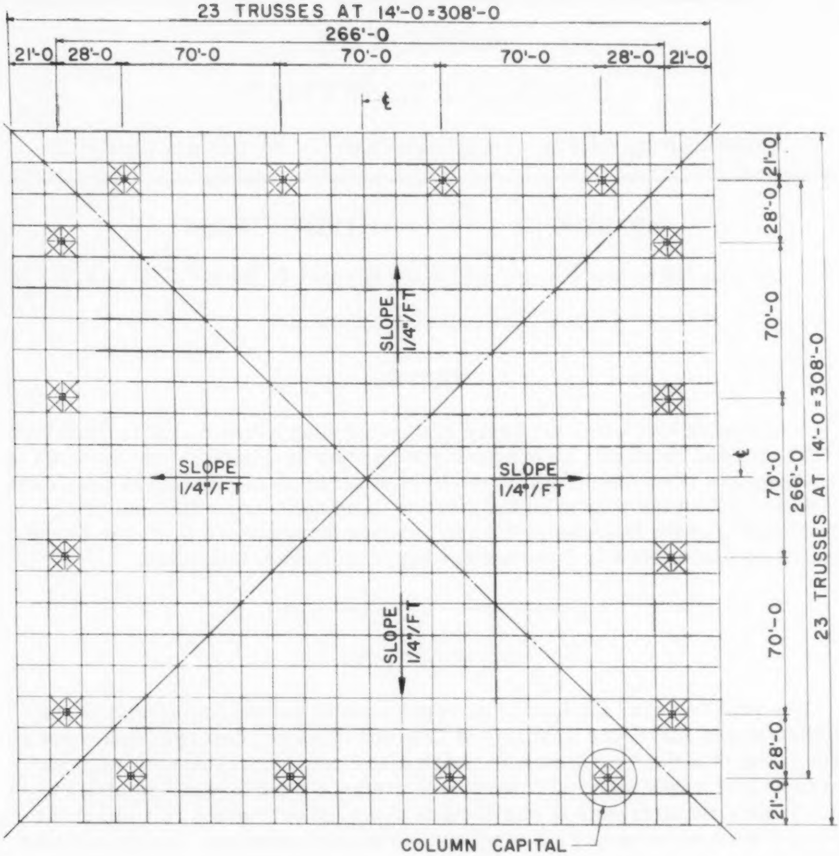
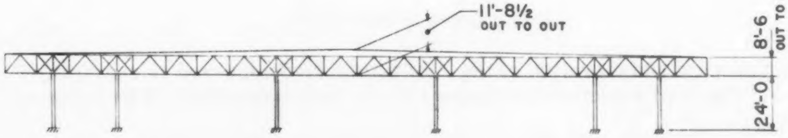


PLAN VIEW



ELEVATION

FIG. 1 PLAN VIEW AND ELEVATION OF ROOF STRUCTURE

the centerline of exterior trusses. There are 23 Warren trusses in each direction spaced 14'-0" on centers which are welded at their points of intersection to provide full continuity in each direction. The trusses are fabricated with structural tees as chords and angles as diagonals and verticals. All fabrication, both shop and field, is by welding.

The roof is supported on 16 columns spaced 70'-0" o.c. and set in 21'-0" from the perimeter, thus providing a clear interior area 266' x 266' square. The columns are located midway between the centerlines of trusses and the load is transferred from the top chords of the four surrounding trusses to a column by means of a "capital" fabricated from standard structural shapes. A photograph of this capital is shown in Fig. 2. Load is transferred from the base of the capital to the column through a stainless steel hemisphere. The stainless steel hemisphere serves as a momentless connection at the top of the column while the base of the column is rigidly fixed to the concrete substructure.

The total depth of the trusses is 8'-6" at the perimeter. A pitch of one-quarter inch per foot was given to the roof surface in four directions in order to gain additional depth at the center and to facilitate drainage. Thus the total depth of the trusses at the center of the roof increases to 11'-8-1/2". The average depth of the truss system is approximately 1/27th of the clear interior span.

A metal deck covered with insulation and a built up roofing forms the surfacing. The perimeter of the roof is sheathed with an aluminum facia. Prefabricated aluminum panels are inserted between the lower chords of the trusses to form an acoustical ceiling.

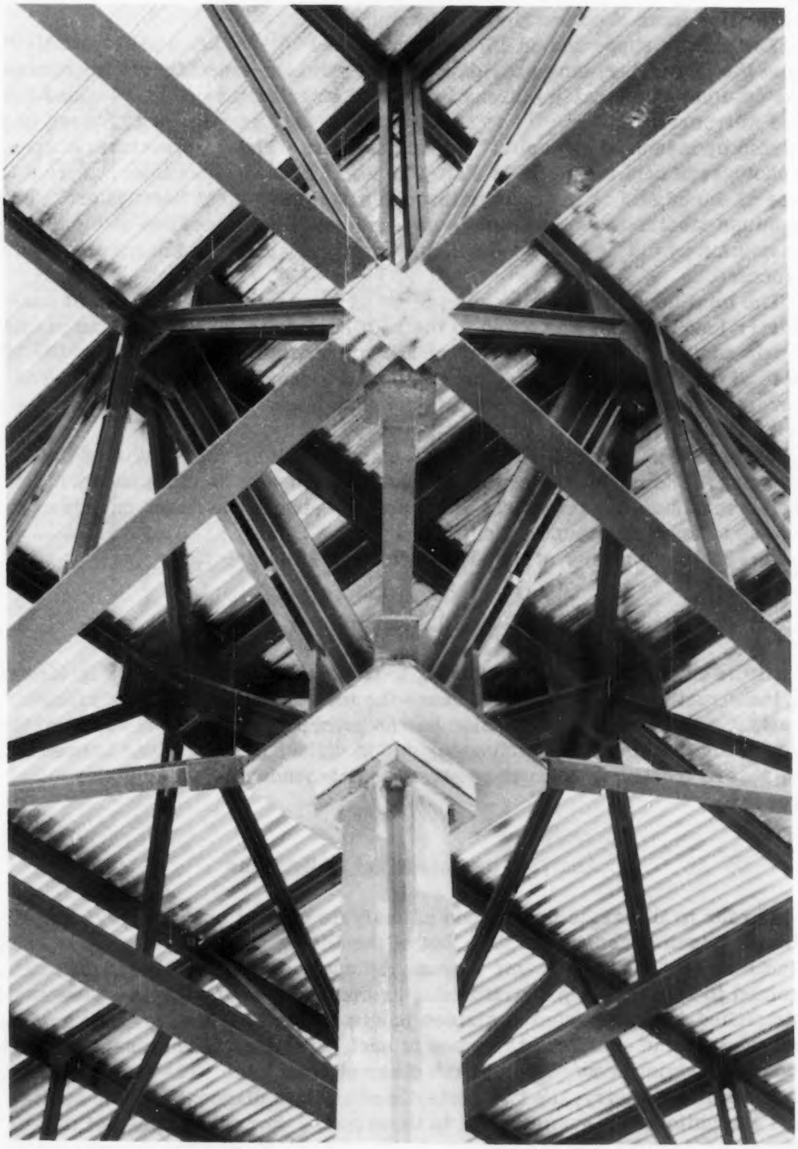
Fig. 26 shows a view of the structure after steel erection.

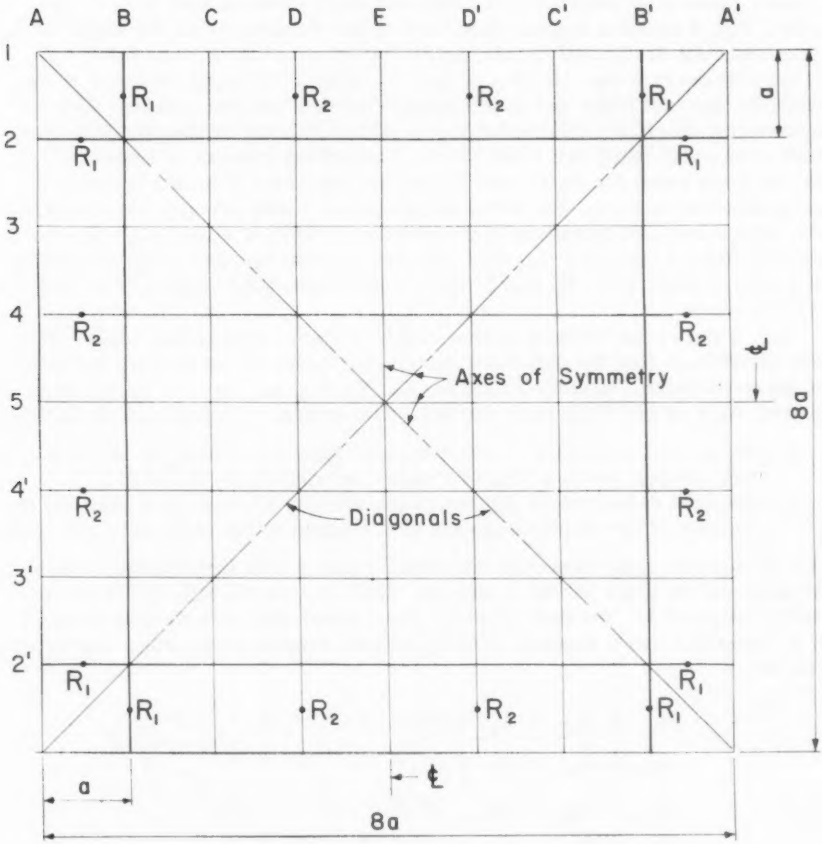
General Assumptions

All the usual assumptions applying to truss analysis were made in idealizing the structure. In addition, because the torsional rigidity of the trusses is small, it was assumed that torsion has no primary effect on the action of the grid. Thus the stresses which occur due to differential rotations between joints were treated as secondary stresses independent of the primary stresses.

Analysis by Consistent Deflections

In order to illustrate the method of analysis without unnecessary complications a much coarser grid than that of the Dining Hall roof (Fig. 8) is used in the following. For the same reason, the applied dead and live loads are assumed to be uniformly distributed over the entire roof. Fig. 3 shows such a simplified grid supported at sixteen points marked R_1 and R_2. The grid is composed of nine equidistant trusses in each direction. The trusses are continuous and rigidly connected to each other at each joint. The structure has four axes of symmetry, the two centerlines and two diagonals, if the applied loads are uniform or symmetrical to these axes. For the purpose of this analysis we assume that the trusses on lines 2, 2', B, B' (Fig. 3), will act as elastic supports for all the other trusses. The uniform load w is converted to concentrated loads applied at the joints. The intersecting trusses act together in supporting the panel loads P but do not share these loads equally





Loads applied on grid:

At interior truss intersections = P
At exterior truss intersections = P/2
At corner truss intersections = P/4
Where: $P = wa^2$
w = uniform load per sq. ft.
a = grid spacing
R_1 & R_2 = column reactions

FIG. 3 GRID TO ILLUSTRATE METHOD OF CONSISTENT DEFLECTIONS

except at the joints along the two diagonal axes of the structure. As a consequence there is an internal force between the trusses at each joint on the grid. Fig. 4 shows a quarter plan view of the structure with the supports R_2 removed. As designated in this figure, we assume that at each joint, one truss will carry a load (1/2P + K) and the other (1/2P - K), where K is the unknown internal force due to interaction between the two trusses. Due to symmetry, there are no internal forces at joints lying on the diagonal axes, hence the panel loads are 1/2P. In the case of the trusses on lines 2, 2', B, B' the loads equal the panel load P plus the reactions from the trusses perpendicular to them. Since the structure and loads are symmetrical about the centerlines and diagonals of the structure, there are only six unknown internal forces (see Fig. 5). The unknown internal forces r_1, r_2, r_3, and r_4, at joints on lines 2, 2', B, and B' have been expressed in terms of K_1, K_2, K_3, K_4, K_5 and K_6.

Fig. 5 shows the trusses as free bodies with the appropriate loads. It is now possible to find the deflection for all the points on the trusses in terms of the panel loads P and the unknowns K_1, K_2, K_3, K_4, K_5, and K_6 by means of virtual work or any equivalent method. The symbol convention is as follows:

δ = the partial deflection of any one panel point on a truss, or the deflection with respect to its points of support on truss 2, 2', B, or B'.

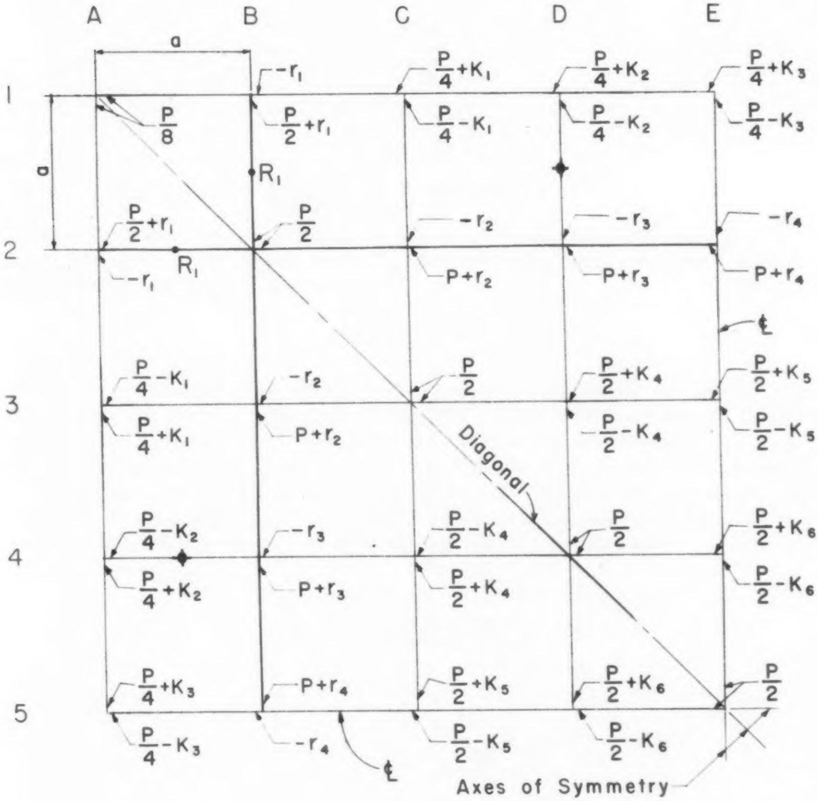
Δ = the total deflection of any one panel point on a truss, or δ plus the deflection of the elastic supports with respect to the stationary points R_1.

The subscripts associated with the terms δ and Δ will indicate the truss and the point on the truss in that sequence. Thus Δ_{1A} is the total deflection of truss 1 at point A. The deflection for the various points in an area bounded by a centerline and a diagonal of the grid with respect to R_1 (total deflections) will be:

$$\begin{aligned}
\Delta_{1A} &= \Delta_{B1} + \delta_{1A} = f_1(P, K_1, K_2, K_3, K_4, K_5, K_6),\\
\Delta_{1C} &= \Delta_{B1} + \delta_{1C} = f_2(P, K_1, K_2, K_3, K_4, K_5, K_6),\\
\Delta_{1D} &= \Delta_{B1} + \delta_{1D} = f_3(P, K_1, K_2, K_3, K_4, K_5, K_6),\\
\Delta_{1E} &= \Delta_{B1} + \delta_{1E} = f_4(P, K_1, K_2, K_3, K_4, K_5, K_6),\\
\Delta_{3C} &= \Delta_{B3} + \delta_{3C} = f_5(P, K_1, K_2, K_3, K_4, K_5, K_6),\\
\Delta_{3D} &= \Delta_{B3} + \delta_{3D} = f_6(P, K_1, K_2, K_3, K_4, K_5, K_6),\\
\Delta_{3E} &= \Delta_{B3} + \delta_{3E} = f_7(P, K_1, K_2, K_3, K_4, K_5, K_6),\\
\Delta_{4D} &= \Delta_{B4} + \delta_{4D} = f_8(P, K_1, K_2, K_3, K_4, K_5, K_6),\\
\Delta_{4E} &= \Delta_{B4} + \delta_{4E} = f_9(P, K_1, K_2, K_3, K_4, K_5, K_6),\\
\Delta_{5E} &= \Delta_{B5} + \delta_{5E} = f_{10}(P, K_1, K_2, K_3, K_4, K_5, K_6).
\end{aligned} \tag{1}$$

If the following deflections are equated:

$$\begin{aligned}
&\Delta_{1C} = \Delta_{C1} \;;\quad \Delta_{1E} = \Delta_{E1} \;;\quad \Delta_{3E} = \Delta_{E3} \;;\\
&\Delta_{1D} = \Delta_{D1} \;;\quad \Delta_{3D} = \Delta_{D3} \;;\quad \Delta_{4E} = \Delta_{E4} \;,
\end{aligned} \tag{2}$$



$P = wa^2$

w = load per sq. ft.

a = grid spacing.

$K_1, K_2, K_3, K_4, K_5, K_6$ = unknown internal forces due to interaction between trusses.

r_1, r_2, r_3, r_4 = reactions at elastic support. (see Fig. 5 for values)

R_1 = column reaction = 8P

✦ = location of columns R_2. (omitted in first stage of analysis)

FIG. 4 QUARTER PLAN OF GRID WITH PANEL LOADS

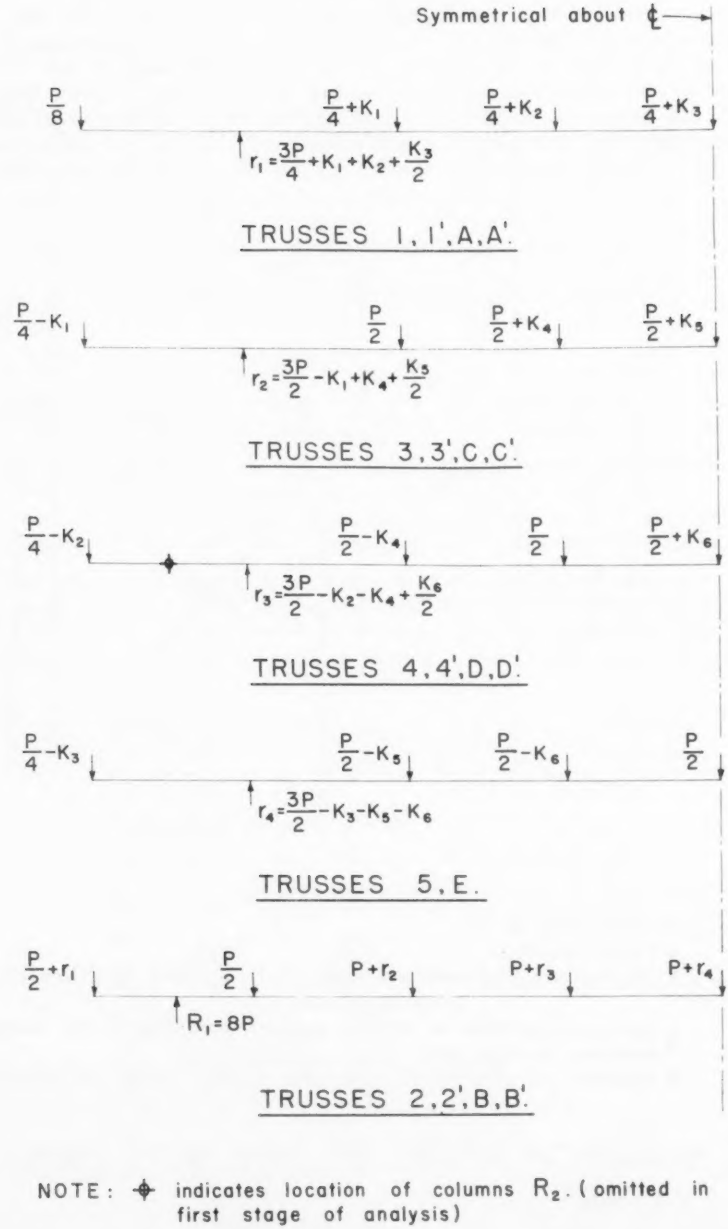


FIG. 5 FREE BODY DIAGRAMS FOR TRUSSES IN FIG. 4

six equations of the following type are obtained:

$$
\begin{aligned}
F_1(P, K_1, K_2, K_3, K_4, K_5, K_6) &= 0,\\
F_2(P, K_1, K_2, K_3, K_4, K_5, K_6) &= 0,\\
F_3(P, K_1, K_2, K_3, K_4, K_5, K_6) &= 0,\\
F_4(P, K_1, K_2, K_3, K_4, K_5, K_6) &= 0,\\
F_5(P, K_1, K_2, K_3, K_4, K_5, K_6) &= 0,\\
F_6(P, K_1, K_2, K_3, K_4, K_5, K_6) &= 0.
\end{aligned}
\tag{3}
$$

Solving these six equations simultaneously yields the six unknown internal forces in terms of the panel loads P. The deflection at the points of application of the other support R_2 is:

$$\Delta_{R_2} = C_1 P. \tag{4}$$

The second stage of the analysis involves the introduction of reactions R_2. Fig. 6 shows a quarter plan view of the grid loaded with the unknown reactions R_2. The loads at the joints consist of the unknown interactions K'_1, K'_2, K'_3, K'_4, K'_5, and K'_6. In Fig. 7 the trusses are represented as free bodies with the appropriate loads. Deflections are computed in the same manner as in stage one (Eqs. 1). If the deflections are equated for the same points as before (Eqs. 2), the six equations obtained will be of the following type:

$$
\begin{aligned}
G_1(R_2, K'_1, K'_2, K'_3, K'_4, K'_5, K'_6) &= 0,\\
G_2(R_2, K'_1, K'_2, K'_3, K'_4, K'_5, K'_6) &= 0,\\
G_3(R_2, K'_1, K'_2, K'_3, K'_4, K'_5, K'_6) &= 0,\\
G_4(R_2, K'_1, K'_2, K'_3, K'_4, K'_5, K'_6) &= 0,\\
G_5(R_2, K'_1, K'_2, K'_3, K'_4, K'_5, K'_6) &= 0,\\
G_6(R_2, K'_1, K'_2, K'_3, K'_4, K'_5, K'_6) &= 0.
\end{aligned}
\tag{5}
$$

Solving these six equations simultaneously yields the six unknown internal forces in terms of the unknown reactions R_2. The deflection at the point of application of R_2 is:

$$\Delta'_{R_2} = C_2 R_2. \tag{6}$$

By equating (4) and (6)

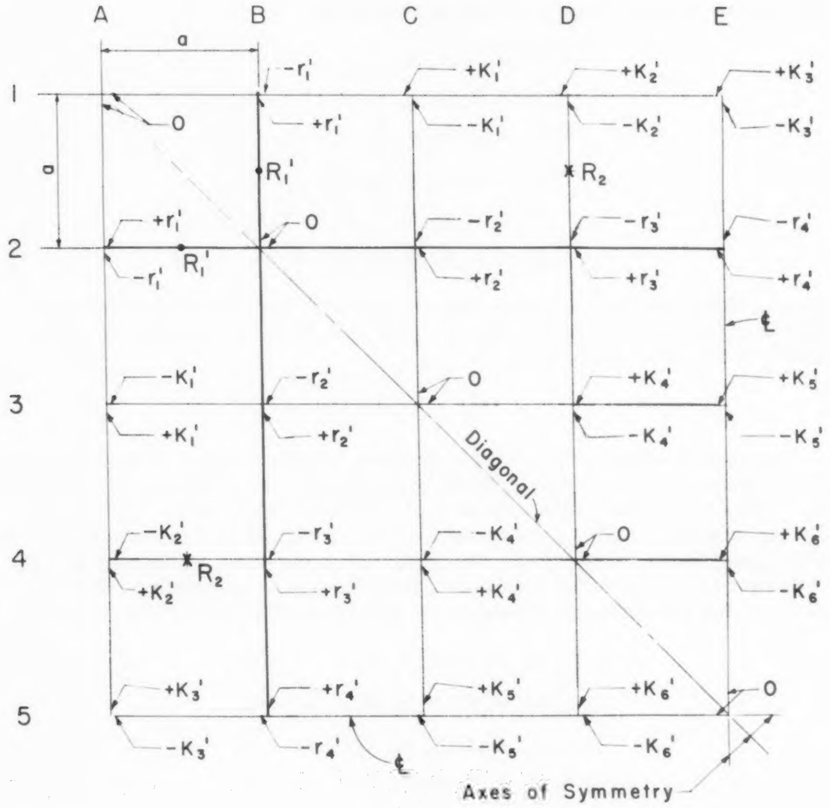
$$\Delta_{R_2} = \Delta'_{R_2}, \tag{7}$$

reaction R_2 is

$$R_2 = \frac{C_1}{C_2} P. \tag{8}$$

The values of K'_1, K'_2, K'_3, K'_4, K'_5, and K'_6 are then also converted to panel loads in terms of P.

The trusses may now be designed by applying the known loads, P, K_1, K_2, K_3, K_4, K_5, K_6, and K'_1, K'_2, K'_3, K'_4, K'_5, and K'_6.



a = grid spacing.
$K_1', K_2', K_3', K_4', K_5', K_6'$ = unknown internal forces due to interaction between trusses.
r_1', r_2', r_3', r_4' = reactions at elastic support. (see Fig. 7 for values)
R_1' = column reaction = R_2
R_2 = loads on grid.

FIG. 6 PANEL LOADS DUE TO REACTION R_2

It should be noted that the matrix of the coefficients of Eqs. (3) and (5) are symmetric.

Analysis As An Equivalent Flexural Grid

A truss carries load in much the same manner as a beam, except for the difference in the method of transmitting shear forces. The result of this

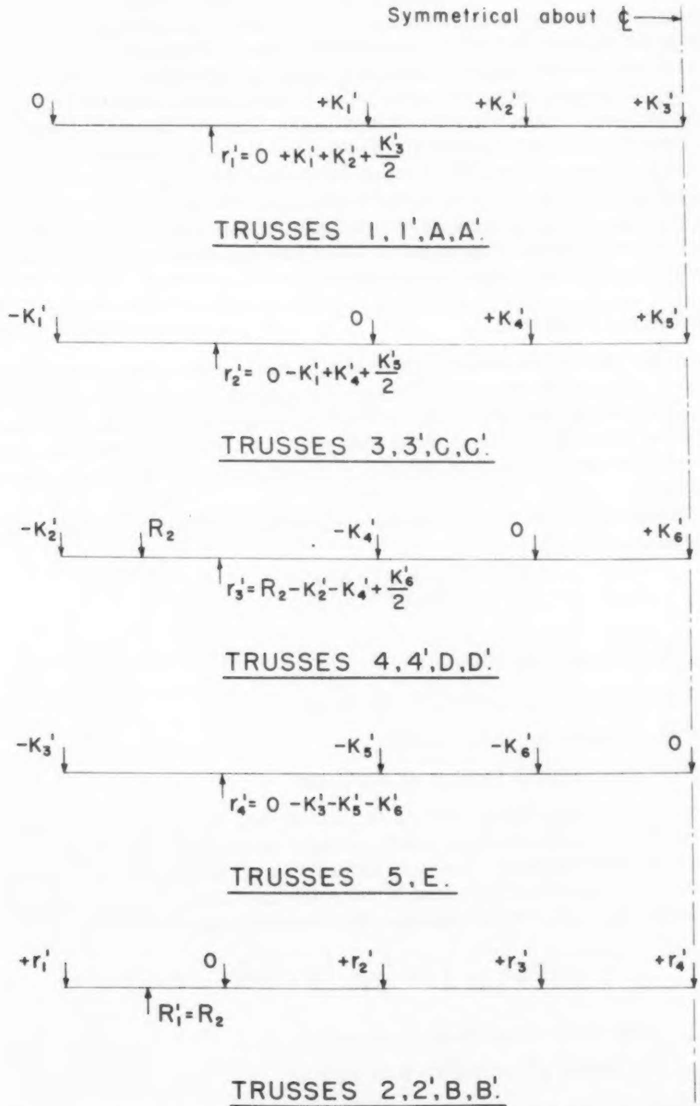


FIG. 7 FREE BODY DIAGRAMS FOR TRUSSES IN FIG. 6

difference is that the shear deflection in a truss of ordinary proportions will usually account for a greater percentage of the total deflection than it would in the case of a beam of ordinary proportions. Shear deflections are generally neglected in beams because they only account for a small percentage of the total deflection and are less significant than the uncertainty in the magnitude of loads, properties of materials, manufacturing tolerances, etc., unless the member is very deep or subjected to some unusual loading. If the depth to span ratio of a parallel chord truss is decreased, the proportion of the total deflection due to shear also decreases. The type of structure considered in this paper has a low depth to span ratio and the shear deformation becomes small enough so that for practical purposes it may be neglected for the same reasons it is neglected in an ordinary beam. Thus the load carried by a very shallow two way truss system can be considered as an equivalent flexural grid and analyzed by numerical techniques.

General Basis of Method

Consider the trusses of the structure to be an orthogonal grid of flexural members, of varying structural properties, which are rigidly connected at their points of intersection. The structural properties of this grid must be determined from the system which they replace. For this analysis members of the grid are subjected to bending and shear only with torsional effects neglected.

The linearized differential equation for a single flexural member (in an X direction) is

$$\frac{d^2}{d X^2}\left(E\, I(x)\frac{d^2 w}{d X^2}\right) = p(x) \tag{9}$$

where E = Youngs modulus

$I(x)$ = Moment of inertia at any point

$w(x)$ = Deflection at any point

$p(x)$ = Intensity of loading at any point

When a grid of orthogonal members is considered, the load $p(x,y)$ along the members, is carried in part by each member where they intersect and by an individual member between points of intersection. If the grid is referred to a cartesian set of coordinates X and Y, with the members running in the X and Y directions, the differential equation for the grid takes the form

$$\frac{\partial^2}{\partial X^2}\left(E I_x \frac{d^2 w}{d X^2}\right) + \frac{\partial^2}{\partial Y^2}\left(E I_Y \frac{d^2 w}{d Y^2}\right) = p(X,Y) \tag{10}$$

where I_x = Moment of inertia in X direction

I_y = Moment of inertia in Y direction

The members of this grid may be considered to have zero flexural rigidity in a direction perpendicular to their length. It can be seen that the terms in parentheses in the above equations are the usual differential expressions for M_x and M_y where M_x and M_y are the bending moments in the X or Y direction at any point on the grid. Thus the equation could also be written as

$$\frac{\partial^2}{\partial X^2}\left(-M_X\right) + \frac{\partial^2}{\partial Y^2}\left(-M_Y\right) = p(X,Y) \tag{11}$$

A convenient solution of this differential equation for a particular set of boundary conditions is possible through the application of the finite difference procedure. The finite difference procedure will ultimately reduce the problem to the solution of a system of linear simultaneous algebraic equations which will yield deflections at particular points when solved. From these deflections the moments, shears, and reactions of the grid can be calculated and the calculated values used to determine the bar stresses in the truss system. It is necessary to write one equation for each point at which the deflection is to be found. The number of points which must be considered is determined by the type of loading, the variations in structural properties, and by considerations of accuracy. For the Dining Hall the deflections at grid intersection points and the column capital support points were sufficient to satisfy these requirements.

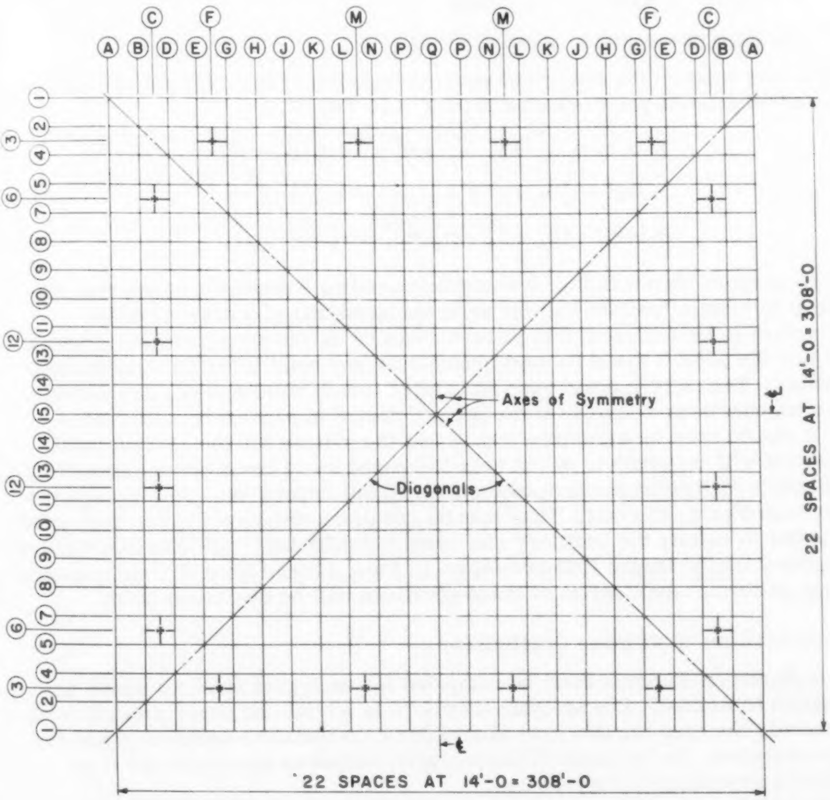


FIG. 8 GRID NOTATION FOR DINING HALL ROOF

Difference Equations

Derivation of the difference equations will not be given in this paper. Information pertaining to the derivation of these equations can be found in reference (1, 2, 3, 4). In order to discuss the finite difference equations, a system of notation is required. Such a system of notation is shown in Fig. 9. The point to which an equation applies is designated as "0". Points surrounding "0" are designated as shown in Fig. 9. When referring to a particular point the point location is indicated by a subscript. Furthermore when a quantity applies to a particular direction, the direction is indicated by a superscript such as M_0^X, the bending moment in the X direction at point 0. Some further notation is as follows:

w = Deflection at a point indicated by a subscript

h = Spacing of trusses or main grid members

I = Moment of inertia

B = $(EI)/h^3$ = Stiffness coefficient

p = Load concentrated at point "0".

With this notation the difference equation solution of the differential equation for some general point (such as point 7-K in Fig. 8) is

$$\begin{aligned} &B_4^X w_8 + B_2^X w_6 + B_1^Y w_5 + B_3^Y w_7 - 2(B_0^X - B_4^X) w_4 \\ &-2(B_0^X + B_2^X) w_2 - 2(B_0^Y + B_1^Y) w_1 - 2(B_0^Y + B_3^Y) w_3 \\ &+ (4B_0^X + 4B_0^Y + B_4^X + B_2^X + B_1^Y + B_3^Y) w_0 = p \end{aligned} \tag{12}$$

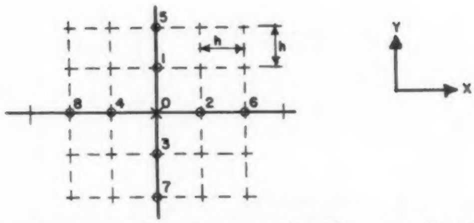
If the notation shown in Fig. 9 would be placed on a sheet of transparent material to form a "pattern" whose scale matched that of a drawing of the structure to be analyzed, this pattern could be moved about over the drawing so that the point 0 would be superposed over any point to which this equation applies. The pattern could even be rotated (which will be necessary for some special equations to follow) if the proper change in superscripts is made.

It can be seen by examining Fig. 9 that the pattern for the general point equation will not apply to points near the boundary of the structure or around the points of support because on the pattern fall beyond the grid edge or over subdivided grid intervals. Thus special equations are required for such points in order to satisfy the boundary and support conditions. The special equations required for the Dining Hall are shown in Figs. 10 thru 21 with their accompanying patterns. Application of these equations will be discussed later.

Determination of Stiffness Coefficients

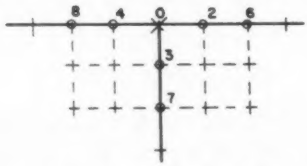
Stiffness coefficients must be computed for each grid point for which an equation is written. The analysis therefore is a trial and error procedure as is usually the case for this type of structure. A tentative design is made and then analyzed. On the basis of the analysis, revisions are made and if required a new analysis carried out.

The stiffness coefficients involve three terms, E. I. and h. Of those three terms only I is a function of position on the structure. The value of I is computed from the properties of the chords of the trusses. Using the properties of the chord members, the moment of inertia I is computed from



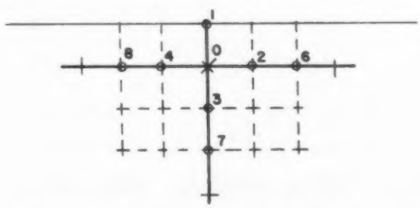
$$B_4^X w_8 + B_2^X w_6 + B_1^Y w_5 + B_3^Y w_7 - 2(B_0^X + B_4^X) w_4 - 2(B_0^X + B_2^X) w_2 - 2(B_0^Y + B_1^Y) w_1$$
$$- 2(B_0^Y + B_3^Y) w_3 + (4B_0^X + 4B_0^Y + B_4^X + B_2^X + B_1^Y + B_3^Y) w_0 = p$$

FIG. 9 GENERAL INTERIOR POINT EQUATION



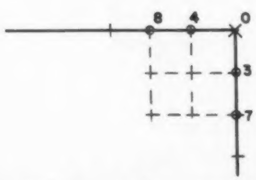
$$B_4^X w_8 + B_2^X w_6 + B_3^Y w_7 - 2(B_0^X + B_2^X) w_2 - 2(B_0^X + B_4^X) w_4 - 2B_3^Y w_3$$
$$+ (4B_0^X + B_3^Y + B_4^X + B_2^X) w_0 = p$$

FIG. 10 GENERAL FREE EDGE POINT



$$B_4^X w_8 + B_2^X w_6 + B_3^Y w_7 - 2(B_0^X + B_4^X) w_4 - 2(B_0^X + B_2^X) w_2 - 2(B_0^Y + B_3^Y) w_3$$
$$- 2B_0^Y w_1 + (4B_0^X + 4B_0^Y + B_4^X + B_2^X + B_3^Y) w_0 = p$$

FIG. 11 GENERAL POINT ADJACENT TO A FREE EDGE

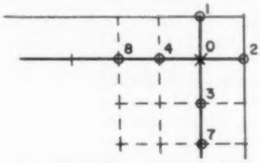


$$B_4^X w_8 + B_3^Y w_7 - 2B_4^X w_4 - 2B_3^Y w_3 + (B_4^X + B_3^Y) w_0 = p$$

FIG. 12 FREE EDGE CORNER POINT

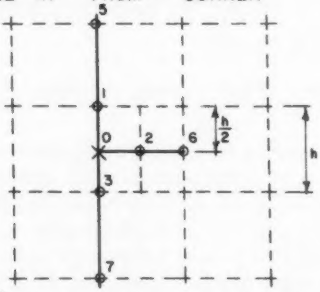
$$B_4^X w_8 + B_3^Y w_7 - 2(B_0^X + B_4^X) w_4 - 2B_0^X w_2 - 2B_3^Y w_3 + (4B_0^X + B_4^X + B_3^Y) w_0 = p$$

FIG. 13 FREE EDGE POINT ADJACENT TO CORNER



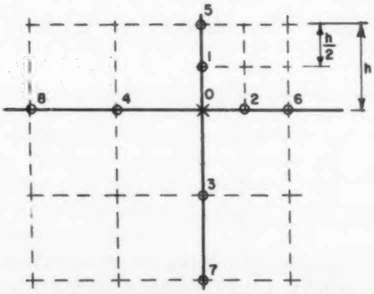
$$B_4^X w_8 + B_3^Y w_7 - 2(B_0^X + B_4^X) w_4 - 2(B_0^Y + B_3^Y) w_3 - 2B_0^Y w_1 - 2B_0^X w_2 + (4B_0^X + 4B_0^Y + B_4^X + B_3^Y) w_0 = p$$

FIG. 14 POINT ONE-IN FROM CORNER



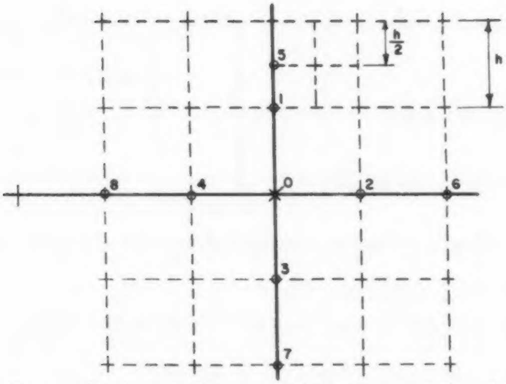
$$8B_2^X w_6 + \frac{8}{3}B_3^Y w_7 + \frac{8}{3}B_1^Y w_5 - 16B_2^X w_6 - (16B_0^Y + 8B_1^Y) w_1 - (16B_0^Y + 8B_3^Y) w_3 + (32B_0^Y + 8B_2^X + \frac{16}{3}B_1^Y + \frac{16}{3}B_3^Y) w_0 = p$$

FIG. 15 SPECIAL EQUATION FOR SUBDIVIDED GRID



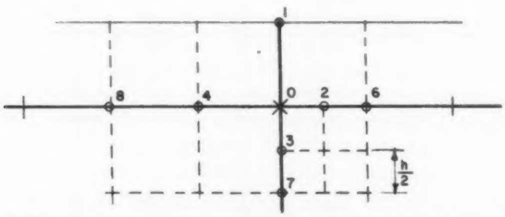
$$B_4^X w_8 + 8B_2^X w_6 + B_3^Y w_7 + 8B_1^Y w_5 - (4B_0^X + 2B_4^X) w_4 - (8B_0^X + 16B_2^X) w_2 - (8B_0^Y + 16B_1^Y) w_1 - (4B_0^Y + 2B_3^Y) w_3 + (12B_0^X + 12B_0^Y + B_4^X + 8B_2^X + B_3^Y + 8B_1^Y) w_0 = p$$

FIG. 16 SUBDIVIDED GRID EQUATION



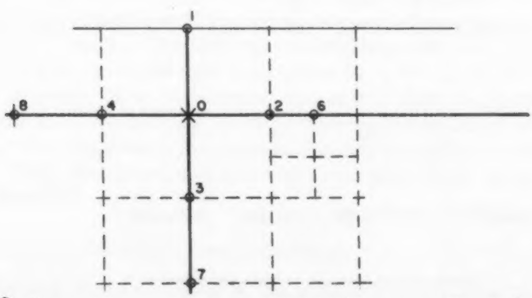
$$B_4^X w_8 + B_2^X w_6 + \frac{8}{3} B_1^Y w_5 + B_3^Y w_7 - 2(B_0^X + B_4^X) w_4 - 2(B_0^X + B_2^X) w_2$$
$$-2(B_0^Y + B_1^Y) w_1 - 2(B_0^Y + B_3^Y) w_3 + (4B_0^X + 4B_0^Y + B_4^X + B_2^X + B_3^Y + \frac{4}{3} B_1^Y) w_0 = p$$

FIG. 17 SUBDIVIDED GRID EQUATION



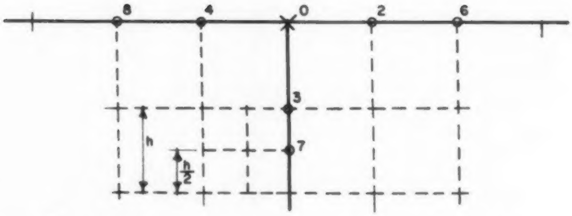
$$8B_3^Y w_7 + 8B_2^X w_6 + B_4^X w_8 - 4B_0^Y w_1 - (8B_0^Y + 16B_3^Y) w_3 - (8B_0^X + 16B_2^X) w_2$$
$$-(4B_0^X + 2B_4^X) w_4 + (12B_0^X + 12B_0^Y + 8B_3^Y + B_4^X + 8B_2^X) w_0 = p$$

FIG. 18 SUBDIVIDED GRID NEAR A FREE EDGE



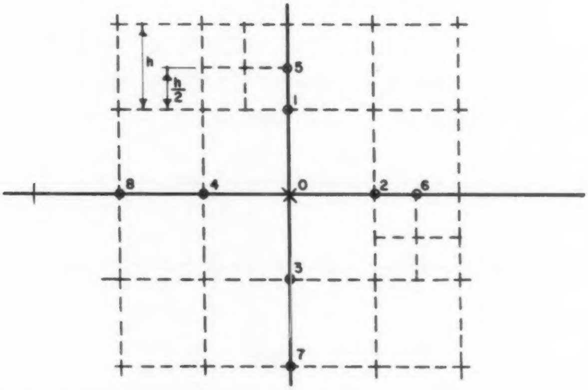
$$B_3^Y w_7 + \frac{8}{3} B_2^X w_6 + B_4^X w_8 - 2B_0^Y w_1 - 2(B_0^Y + B_3^Y) w_3 - (2B_0^X + 4B_2^X) w_2$$
$$-2(B_0^X + B_4^X) w_4 + (4B_0^X + 4B_0^Y + B_3^Y + B_4^X + \frac{4}{3} B_2^X) w_0 = p$$

FIG. 19 SUBDIVIDED GRID NEAR A FREE EDGE



$$B_4^X w_8 + B_2^X w_6 + \frac{8}{3} B_3^Y w_7 - 4B_3^Y w_3 - 2(B_0^X + B_2^X) w_2 - 2(B_0^X + B_4^X) w_4$$
$$+ (4B_0^X + \frac{4}{3} B_2^Y + B_4^X + B_2^X) w_0 = p$$

FIG. 20 FREE EDGE POINT NEAR SUBDIVIDED GRID



$$B_4^X w_8 + \frac{8}{3} B_2^X w_6 + \frac{8}{3} B_1^Y w_5 + B_3^Y w_7 - 2(B_0^X + B_4^X) w_4$$
$$-(2B_0^X + 4B_2^X) w_2 - (2B_0^Y + 4B_1^Y) w_1 - 2(B_0^Y + B_3^Y) w_3$$
$$+(4B_0^X + 4B_0^Y + B_4^X + B_3^Y + \frac{4}{3} B_2^X + \frac{4}{3} B_1^Y) w_0 = p$$

FIG. 21 SUBDIVIDED GRID BETWEEN COLUMN CAPITALS

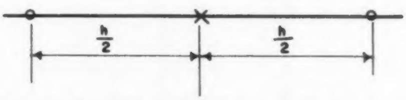


FIG. 22 MOMENT PATTERN — EQUAL SPACES

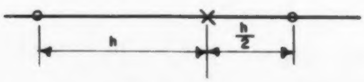


FIG. 23 MOMENT PATTERN — UNEQUAL SPACES

ST 2

$$I = a_T d_T^2 + a_b d_b^2 \tag{13}$$

where a_t = Area of top chord

a_b = Area of bottom chord

d_t = Distance from the center of gravity of both chords to the center of gravity of the top chord

d_b = Distance from the center of gravity of both chords to the center of gravity of the bottom chord

When the area of the chords varies between deflection points an average value of I is used.

The stiffness of the column capitals supporting the roof structure could not be found as described above because of their triangular shape. In order to find the stiffness of the capitals, arbitrary loads were placed at the truss connection points and the resulting deflections computed. These deflections were equated to those of a uniform beam supported in the same manner and an equivalent I computed.

Application of Difference Equations

The points on the structure for which equations will be written must be labeled in some manner. No equations are written for points which do not deflect, such as points of support. If there is any symmetry of support or loading, the number of independent deflection points may also be reduced. When carrying a uniform live load, the Dining Hall roof shown in Fig. 8 has four lines of symmetry, because of the symmetry of support, dead loads and physical properties of the structure. In this case only one eighth of the roof area, consisting of points on and between a centerline and diagonal, will have independent deflections. Assuming the tops of the columns as fixed points there are 86 independent points for which equations must be written. This yields a system of 86 simultaneous equations. These equations were solved on a high speed digital computer.

The equation for each point is obtained by superposing (physically or mentally) the difference "pattern" over the point and substituting the actual point designations for those on the difference equation pattern. Because there are never more than ten non-zero terms in one equation, the equations are relatively simple to write. The writing of equations will be simplified if the stiffness coefficients and loads are multiplied by a common factor to yield numbers close to unity. The deflections which will then be found will be relative values which must be divided by the common factor if actual values are required. After the equations are written the matrix of the coefficients must be symmetric. This condition will provide a partial check of the equations after they are written.

Calculation of Shears, Moments, and Reactions

Moments, shears, and reactions in the grid are calculated from deflection values found by solving the simultaneous equations. The bar stresses are calculated from the moments and shears.

Three cases appear in calculating moments; one where the grid spacing is uniform and equal to "h", and the other two where there is a subdivision of

the grid such as occurs around the column capitals. Consider first the case where the grid has a uniform spacing "h". Using the notation of Fig. 9 where "0" is the point at which the moment is to be calculated, the expression is

$$M_0^x = \frac{EI_0^x}{h^2}(w_4 - 2w_0 + w_2)$$

or

$$M_0^y = \frac{EI_0^y}{h^2}(w_3 - 2w_0 + w_1) \tag{14}$$

The part of the notation in Fig. 9 applying to the moment calculation may be thought of as a transparent moment pattern drawn to an appropriate scale. This moment pattern, which is simply a line with two equal subdivisions, can be moved about over a scale drawing of the grid and is used to calculate the moment at all points where it fits.

Where a grid subdivision exists, the previous pattern will not fit. At such points two patterns are considered one of which has equal divisions of width h/2, and one which has unequal spacings of h and h/2. These patterns and notations are shown in Figs. 22 and 23. With the appropriate superscript added as determined by the direction of the moment, the moment expressions are for equal spacing (Fig. 22).

$$M_0 = 4\frac{EI_0}{h^2}(w_4 - 2w_0 + w_2) \tag{15}$$

and for unequal spacing (Fig. 23).

$$M_0 = \frac{8}{3}\frac{EI_0}{h^2}(w_4 - 3w_0 + 2w_2) \tag{16}$$

If the moment is known at two adjacent points, the shear is simply the difference of the moments divided by the length of the interval. Signs are determined by the usual engineering sign convention. (Positive moment causes compression in top fibers, etc.) Reactions are equal to the sum of the shears about the support plus any tributary load which goes directly to the support.

Summary of the Analysis as an Equivalent Flexural Grid

The analysis of a grid by the finite difference procedure may be summarized as follows:

1. Label the grid so that various deflection points can be easily identified.
2. Calculate and tabulate the loads and stiffness coefficients for each point.
3. Write the system of simultaneous equations by using the appropriate individual difference expressions.
4. Solve the equations on a digital computer or by any other means.
5. Calculate and tabulate the moments, shears, and reactions from which the truss bar stresses can be found.

Comparison of Methods and Discussions of Results

Each method presented has individual advantages and shortcomings. Consistent deflections is a method which conforms closely with conventional methods and assumptions of truss analysis. However, it requires tedious deflection calculations and yields equations which in general do not have zero

terms. The method of an equivalent flexural grid as used here disregards shear deformations, however, this effect is of small significance provided the depth to span ratio is small. Although this method requires more equations than the previous one, the equations are much simpler in form since there are never more than ten non-zero terms in each equation. Unlike the previous method no deflection calculations are necessary. If the supports fall on a line other than the centerline of a truss, the problem is more easily treated by the equivalent flexural grid method. There are some disadvantages common to both methods; numerical errors or errors in signs are not apparent until the equations have been solved; five or six place accuracy is necessary to obtain reliable results; as is common to this type of structure, the analysis is a trial and error procedure, thus it may be necessary to analyze the structure several times before the resulting stresses match the members selected.

In the case of the Dining Hall it was deemed desirable to use both methods of analysis. This procedure would assure a solution free of any errors that might be inherent in either of the two methods and at the same time serve as an independent check. The method of consistent deflections was selected for the initial phase. A complete analysis by the method of consistent deflections would have required 63 equations, a task far beyond the time limits. In order to simplify the calculations the number of equations was reduced to 21 by eliminating trusses on lines E, H, K, P, 5, 8, 10 and 14 (see Fig. 8), and introducing their stiffness and loads in the adjoining trusses. After the equations were solved the stresses were distributed to the trusses which had been eliminated on the basis of their relative stiffness. When this design was completed the structure was analyzed by the equivalent flexural grid method, but in this case equations were written for all the points. The solution of the resulting 86 equations yielded stresses approximating those obtained by the first less exact solution. A comparison of the stresses in the lower chord for selected points is shown in Fig. 24. Fig. 24 indicates that the stresses at members located two or more panel points from any support checked adequately. For members close to the supports where the magnitude of the bending stresses is small, the two solutions compare only qualitatively. However, the magnitude of the stresses is such that member sizes consistent with practical minimum member selection in design, were adequate to carry the stresses resulting from both solutions in most cases. The discrepancy between the two solutions could also be attributed in part to the change in some member sizes on the basis of the first analysis. This altered the relative stiffness between trusses and changed the distribution of stresses.

The anticipated deflection contours for total dead and live loads derived from the final solution are shown in Fig. 25. The maximum calculated total load deflection is approximately 12 inches which is 1/266 of the interior span. The anticipated live load deflection is 5-1/2 inches or 1/580 of the interior span. This is considerably less than the common value of 1/360 often applied to buildings.

The total weight of steel in the superstructure is 1222 tons. A tabulation of the weight of steel required for various parts of the structure and the weight per square foot on the basis of gross roof area is as follows:

Trusses	911 tons	19.2 psf
Sub-framing	42 tons	0.9 psf
Total steel in roof system	953 tons	20.1 psf

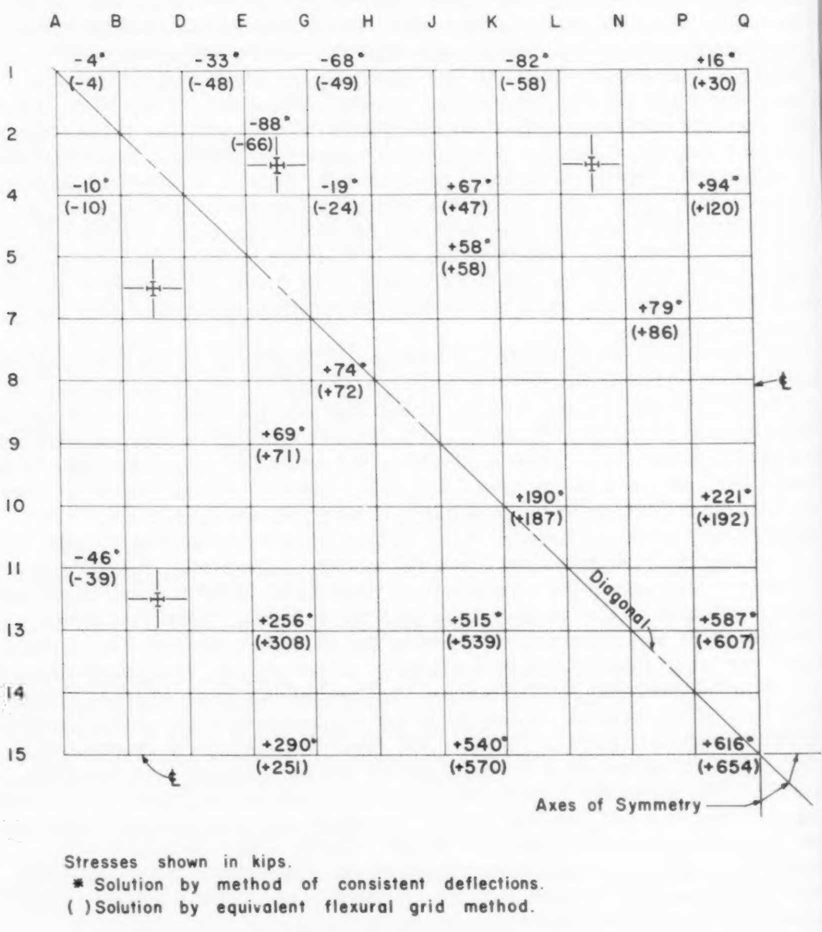
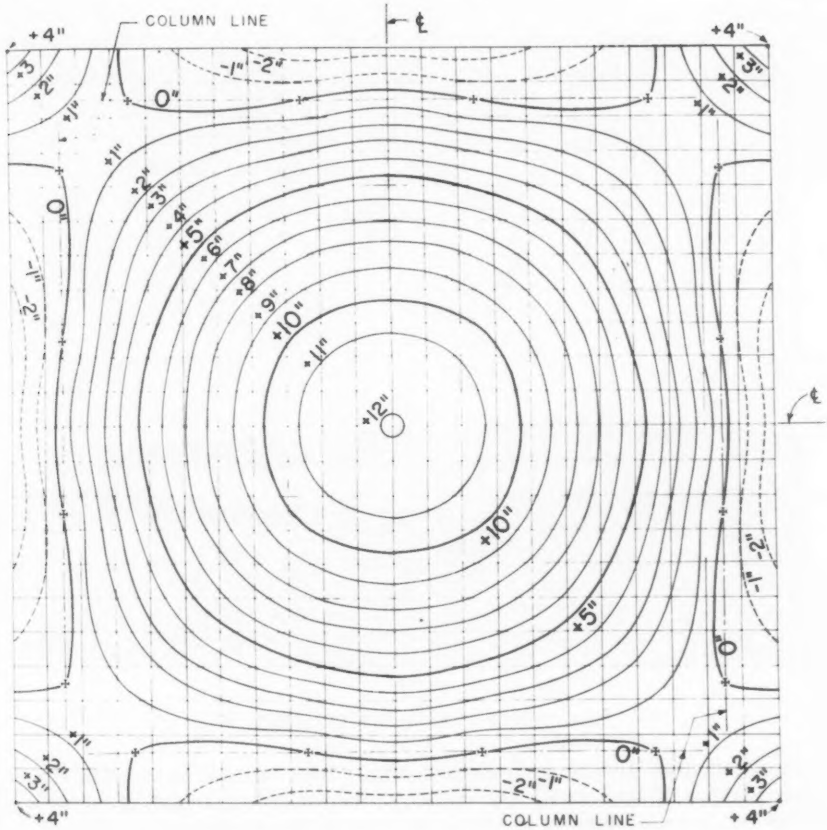


FIG. 24 COMPARISON OF LOWER CHORD STRESSES BY TWO METHODS OF ANALYSIS

Column capitals	119 tons
Columns, base plates, etc.	150 tons
Total	1222 tons

A preliminary design estimate indicated that a one way truss system of similar depth to span ratio would have required approximately 26.0 pounds of steel per square foot or 30 per cent more than the two way system. However, due to increased fabrication costs for the two way system, there was a 15 per cent increase in the bid unit price of steel. Thus a two way system provided an economical structure for the Dining Hall roof.



NOTE: Deflections are for dead load plus maximum uniform live load. Positive deflections are downward. Specified camber is equal to deflections shown but of opposite sign.

FIG. 25 CALCULATED DEFLECTION CONTOURS OF ROOF STRUCTURE

CONCLUSIONS

The use of methods involving large numbers of simultaneous equations is no longer a prohibitive factor in structural analysis because digital computers which will solve these equations are readily accessible to any engineer.

The two methods of analysis presented, consistent deflections or solution with finite differences, yield essentially the same results.

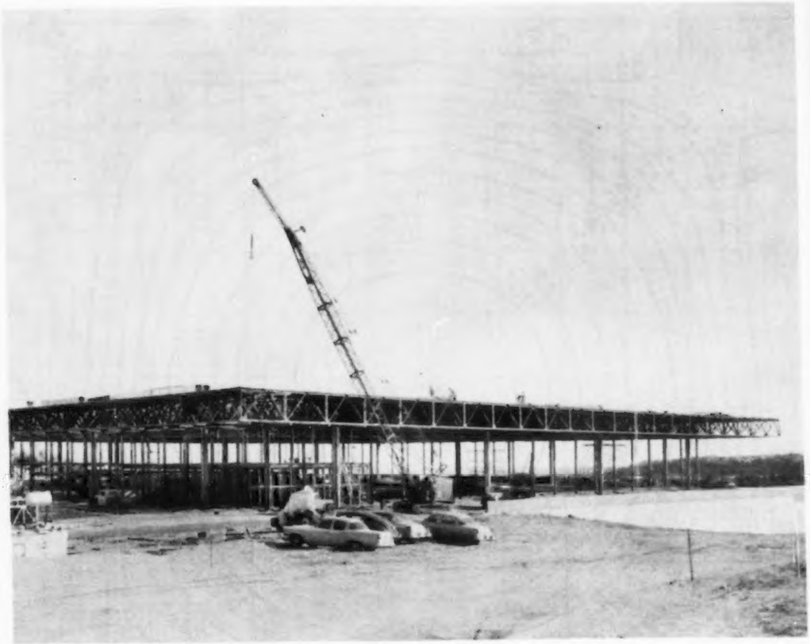


Fig. 26 VIEW OF ROOF STRUCTURE AFTER ERECTION

The equivalent flexural grid method entails less calculation in spite of the larger number of equations. The change of the size of a particular member affects only a small number of terms in a few of the equations, thus facilitating subsequent analysis.

In spite of the small depth to span ratio deflections are not a problem in two way systems.

A two way truss system provided an economical structure for the Dining Hall.

ACKNOWLEDGEMENTS

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REFERENCES

1. Salvadori, M. G., and Baron, M. L., "Numerical Methods in Engineering", Prentice Hall, Inc., New York, 1952.

2. Shaw, F. S., "An Introduction to Relaxation Methods", Dover Publications, Inc., 1953.

3. Southwell, R. V., "Relaxation Methods in Theoretical Physics", Oxford University Press, London, 1946.

4. Southwell, R. V., "Relaxation Methods in Engineering Science", Oxford University Press, London, 1940.

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OVERLOAD FACTORS CAN CAUSE ULTRA-CONSERVATIVE DESIGN

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SYNOPSIS

This paper reviews the use of overload factors in establishing design stresses for reinforced concrete and prestressed concrete structures and for steel transmission towers. It compares this with results of conventional design stress assumptions and finds overload factors create a much higher safety factor than would normally be expected.

Over the years in which the science of structural design has been developing, engineers have experimented with various methods for introducing a factor of safety into the design of structures. These experiments have generally followed two routes in working with the known breaking or yielding strength of the material and the load to be applied to the structure. The common practice incorporated in most codes in the United States has been based on selecting some fraction of the breaking or yielding stress as the allowable design stress to design the structure for the anticipated loads. The other method commonly ignored in this country has been based on using the breaking or yielding stress as the allowable design stress and increasing the anticipated loads by some arbitrary factor to obtain the design load on the structure. This factor is commonly referred to as an "Overload Factor".

The primary argument in favor of the use of overload factors in obtaining the factor of safety has been that this enables the designer to vary the factors of safety for various portions of the load depending upon the nature of the portion of the load and the degree of accuracy to which its magnitude can be estimated.

Until recently, the only major recognized code in the United States advocating the use of overload factor design was the National Electric Safety Code covering the design of electrical transmission towers. Recently, however,

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ACI-ASCE Joint Committees 323 and 327 have incorporated the use of overload factors in their recommendations for the design of prestressed concrete and for the design of conventional reinforced concrete by ultimate strength design. With these two respected committees composed of eminent engineers taking a stand in favor of overload factor design, it seems to be time for structural engineers to evaluate carefully the advantages and disadvantages of the system.

Advocates of overload factor design universally claim that this permits them to safely utilize a much higher design stress than could be done by conventional methods. Transmission line engineers have been convinced of it for years. The same claim is echoed for the use of overload factor design in concrete by ACI-ASCE Committees 323 and 327. In the discussion on the report of Committee 327, discussers and committee alike warned that it meant a "significant decrease in overall factor of safety".

Proponents of this method have argued that the dead load of a structure can be estimated very accurately and hence, in designing for this portion of the load, it is proper to approach quite closely to some limiting stress, such as the yield point or some high percentage of the ultimate stress. It is further argued that the live loads on a structure cannot be predicted as closely and, therefore, in designing for this portion of the load, it is proper to design for a much lower stress. It is claimed that this gives a much more efficient design with a net reduction in factor of safety, which justifies the complications added to the design.

Opponents of this scheme have advanced two (2) arguments. First—the complexities of varying load factors cause the designer to lose sight of the actual stress conditions under the load. Second—the dead load is the one load which the designer KNOWS positively will be on the structure and should have some factor of safety to allow for flaws in the material or workmanship. On the other hand, most live loads and wind loads are selected on the generous side and contain an inherent factor of safety in that these loads are seldom attained.

Leaving for the moment the pros and cons of this particular argument, it is interesting to see how the overload factor method works out in actual practice. As mentioned above, the two (2) major examples of overload factor design in the codes of the United States are the following:

I. Recommendations for the design of reinforced concrete structures by ultimate strength theories. ACI-ASCE Joint Committee No. 327. (Recommendations of ACI-ASCE Committee 323 are similar).

In an excellent paper setting forth formulae for distribution of stresses by ultimate strength theories, the Committee added the requirement that the member should be designed at 85% of the 28 day compressive strength for a load consisting of the larger of the following:

A. $U = 1.2B + 2.4L$
B. $U = K(B + L)$
C. $U = 1.2B + 2.4L + 0.6W$
D. $U = 1.2B + 0.6L + 2.4W$
E. $U - K(B + L + 1/2W)$
F. $U = K(B + 1/2L + W)$

Where "U" is the ultimate load, "B" the basic load or dead load, "L" the live load and "W" the wind load, "K" equals 2 for members subjected to axial

loads and 1.8 for members in flexure only. In the discussion below, "K" has been taken at 1.8.

II. National Electric Safety Code requirements for the design of electrical transmission towers provides that the towers should be designed at the yield point for loads such as the following:

G. Load = 1.27v + 2.54TW + 1.65P

Where "V" is the vertical load, "TW" the load due to transverse wind and "P" the load due to wire pull on the tower in a longitudinal or transverse direction due to a change in direction at the tower or due to deadending the wires.

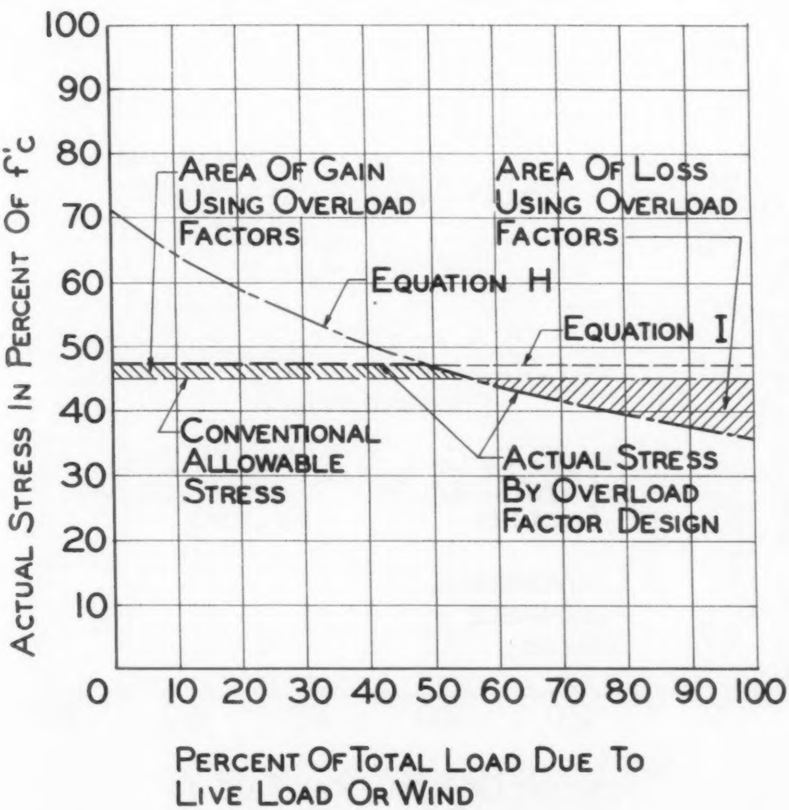

Figure 1

Variations In Actual Stress In Members Using A.C.I. Overload Factors Wind Load = 0 Or Live Load = 0

In designing members under these systems, we can see that the actual stress in the member is:

$$\text{Actual stress} = \frac{\text{actual load}}{\text{design load}} \times \text{design stress}$$

For the concrete design discussed under (I) above, in cases with no wind load, the design load governing will be the larger of the two values of "U" determined by Eqs. A and B and therefore, the actual stress will be the smaller of the following:

$$\text{H. Actual stress} = \frac{B + L}{1.2B + 2.4L} \times 0.85\, f_c'$$

$$\text{I. Actual stress} = \frac{B + L}{1.2(B + L)} \times 0.85\, f_c'$$

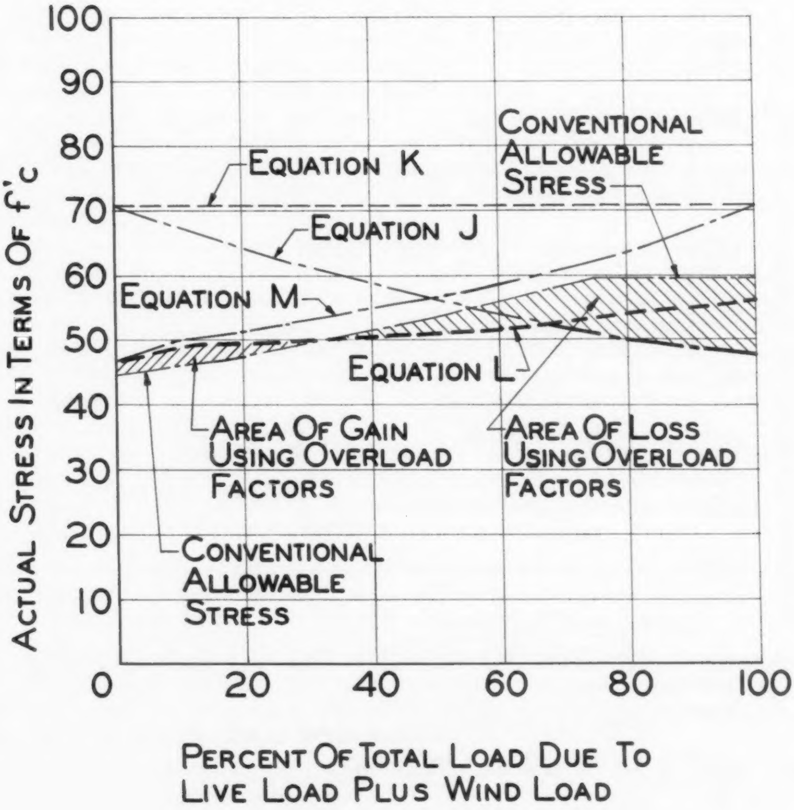

Figure 2

Variations In Actual Stress In Members Using A.C.I. Overload Factors Wind Load = 0.5 Live Load

In Fig. 1, these values are plotted in terms of per cent of f_c' using various ratios of live load to dead load. Also plotted is the allowable stress if conventional design methods had been followed.

Similarly, if wind load and live load must both be considered, the design load governing will be the largest of the four values of "U" determined by equations C, D, E and F, and therefore, the actual stress will be the smallest of the following:

J. Actual stress $= \dfrac{B + L + W}{1.2B + 2.4L + 0.6W} \times 0.85\ f_c'$

K. Actual stress $= \dfrac{B + L + W}{1.2B + 0.6L + 2.4W} \times 0.85\ f_c'$

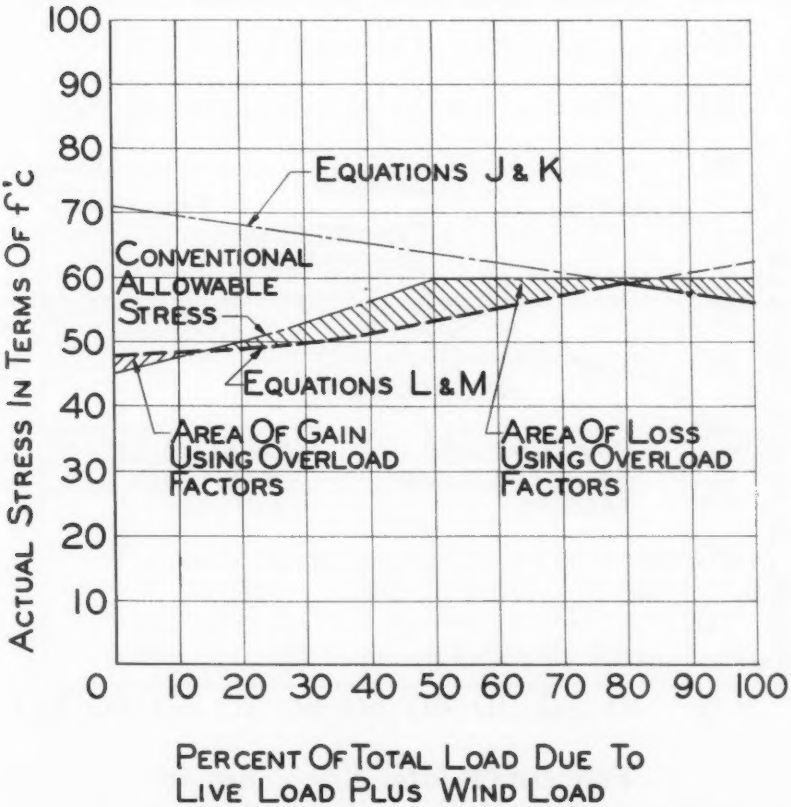

FIGURE 3

VARIATIONS IN ACTUAL STRESS IN MEMBERS USING A.C.I. OVERLOAD FACTORS WIND LOAD = LIVE LOAD

L. Actual stress = $\dfrac{B + L + W}{1.2B \text{ X } 1.2L + 0.6W} \times 0.85\ f_c'$

M. Actual stress = $\dfrac{B + L + W}{1.2B + 0.6W + 1.2L} \times 0.85\ f_c'$

In Figs. 2, 3 and 4, these values are plotted for various ratios of live load to dead load, keeping the wind load as a fixed percentage of the live load in each case. In Fig. 2, wind load is equal to 0.5 times the live load; in Fig. 3, wind load equals live load and in Fig. 4, the wind load is equal to twice the live load.

In Figs. 1 through 4, the areas of gain or loss through the use of overload factor design over conventional design are indicated. In Fig. 5, the relationship of the actual stress for each case to its conventional allowable stress is

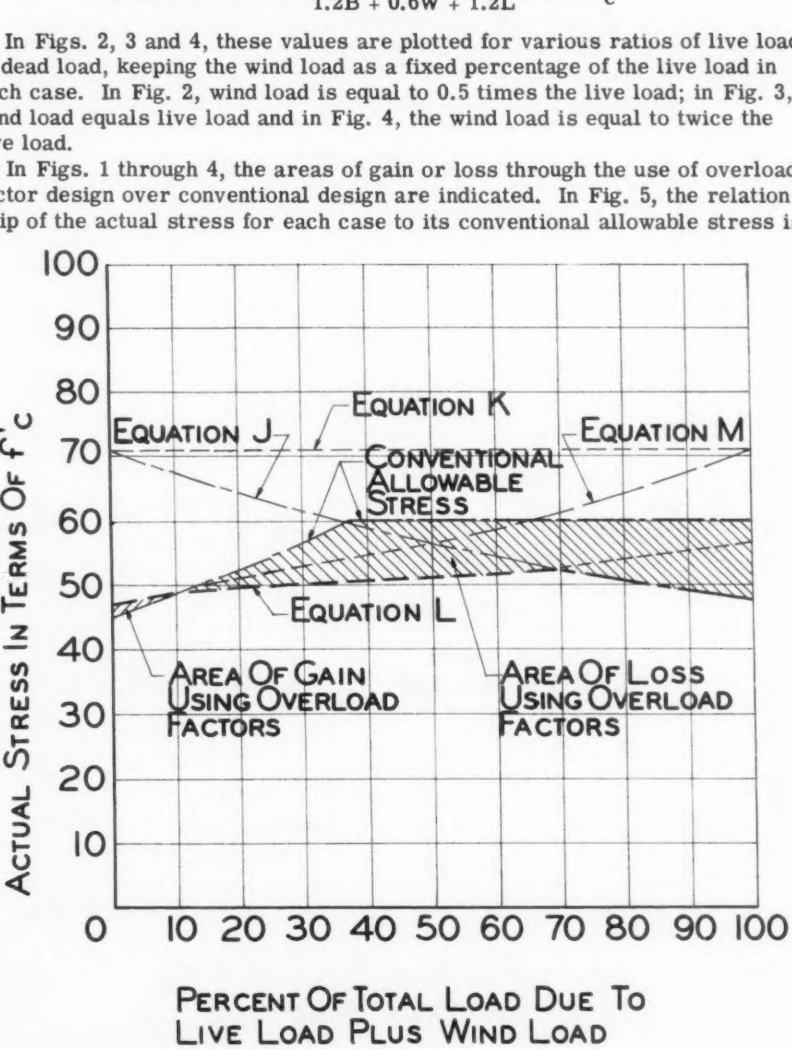

FIGURE 4

VARIATIONS IN ACTUAL STRESS IN MEMBERS USING A.C.I. OVERLOAD FACTORS WIND LOAD = 2.0 LIVE LOAD

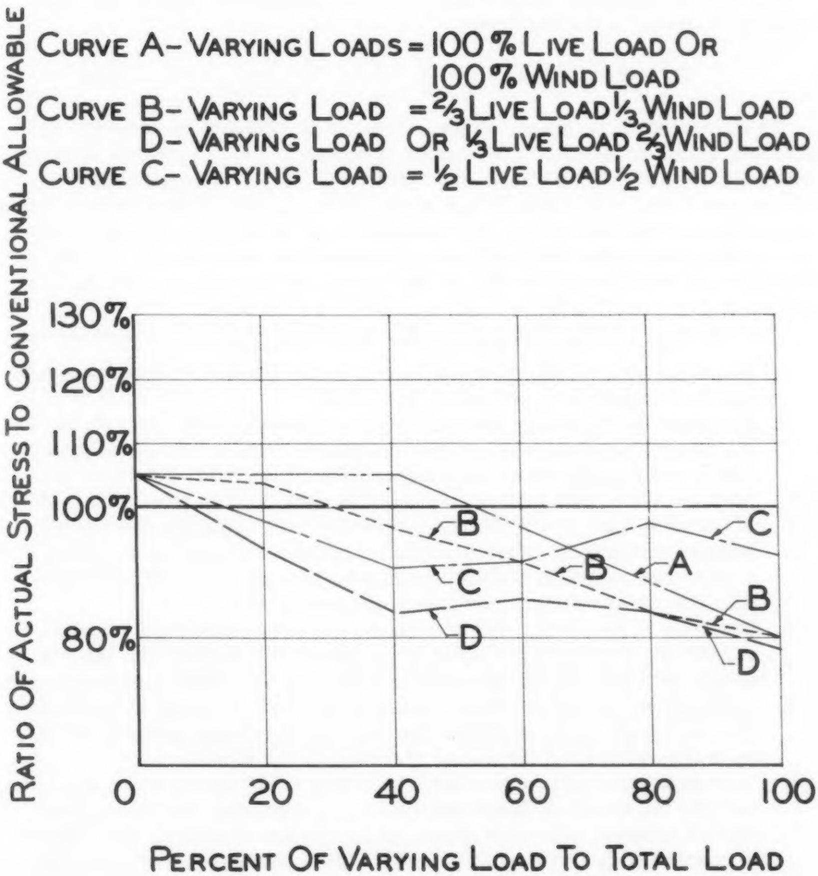

Figure 5

Variations Of Actual Stress In Overload Factor Design With Different Combinations Of Live Load And Wind Load Compared To Conventional Allowable Stress (A.C.I.-318-56)

shown and it is significant that most of these fall below 100%. Thus, except for a small range of values wherein the primary load is dead load, the net effect of the use of the ACI-ASCE overload factors is the use of a lower allowable stress and more conservative structures.

For the transmission tower design criterion discussed under (II) above, the actual stress is the following:

$$\text{N. Actual stress} = \frac{V + TW + P}{1.27V + 2.54TW + 1.65P} \times \text{yield point}$$

By fixing "P" in terms of "V", a curve can be obtained to show the variations in actual stress as "TW" varies in terms of "V". This was done for several values of "P" and the curves plotted in Fig. 6. Surprisingly, these curves are very close together and show that the actual stress on the member varies from about 70% of the yield point down to about 60%. Considering steel with a yield point of 33,000 psi, the use of the conventional design stress of 20,000 psi would be about 60% of the yield point and would meet the other curves at TW = 0.4V.

The following conclusions may be drawn from the preceding discussion:

1. Overload factor design as now in use in the United States does not result in a lower factor of safety generally.
2. Overload factor design for reinforced or prestressed concrete permits slightly higher design stresses than conventional methods for cases where dead load forms a high percentage of the load, but as soon as live load and wind load form an appreciable part (varies from 10% to 50%) of the total load, lower design stresses result requiring the use of heavier structures. This reaches a maximum where the overload factor method can result in a structure twenty-five per cent (25%) heavier than that obtained by conventional methods.
3. Overload factor design for transmission tower work does result in a slightly higher allowable stress than conventional methods, but not nearly so much higher as normally assumed by transmission engineers.
4. Application of overload factor designs incorporate many complicating and confusing issues in the design which prevent the designer from having a true picture of the actual stresses in the structure.
5. Less expensive or comparable structures would result from discarding entirely the overload factor approach and retaining the conventional attitude towards allowable stresses and factor of safety. This should be coupled with a review of present allowable stresses with consideration given to increasing them where improved materials and conditions warrant such a change.

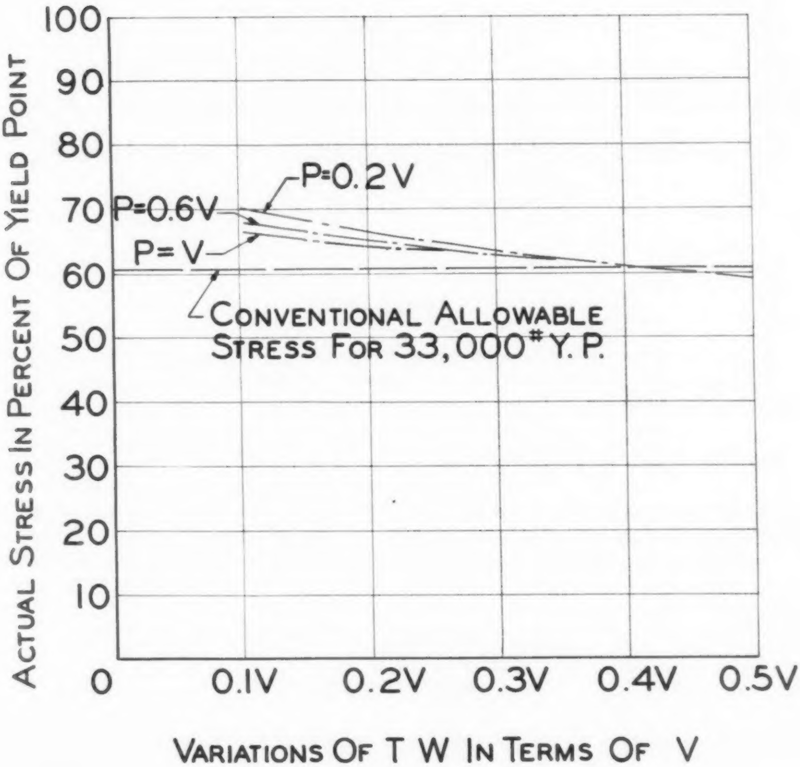

FIGURE 6

VARIATIONS IN ACTUAL STRESS IN MEMBERS USING N.E.S.C. OVERLOAD FACTORS FOR TOTAL DESIGN LOAD ON MEMBERS WITH DESIGN STRESS EQUAL TO YIELD POINT (EQUATION N)

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ASEISMIC DESIGN OF STRUCTURES BY RIGIDITY CRITERION

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SUMMARY

It has become apparent that the earthquake design coefficients of many building codes are insufficient for certain types of structures, particularly for industrial and other, more specialized types. A set of curves is presented here which show design coefficients as function of the rigidity of structures. These coefficients are based on the modified response spectra of one-mass systems obtained by dynamic analysis.

It is hoped that the paper may serve as supplementary reference for the convential aseismic design of special structures and that it will evoke discussion which may lead, ultimately, to the establishment of a generalized seismic design criterion.

INTRODUCTION

The main factors to be considered in a seismic design may be classified as follows:

(1) The nature, pattern, and intensity of dynamic forces produced by earthquake accelerations
(2) The response of various types of structures to these forces
(3) Variables encountered in ground motions and various types of structures.

Because dynamic analysis for any transient force is usually time-consuming and tedious, practically all codes(6,7,8,9,10) provide seismic coefficients by which structures may be designed for equivalent static loads. Unfortunately, numerical values of these coefficients are different in the various

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local codes. It is recognized that most codes are inadequate, as far as industrial and certain other types of structures are concerned. Further, it has been shown by some investigators that the rigidity of the structure governs its response to dynamic forces. The codes, as a rule, do not take this aspect of the structure fully into consideration.

The purpose of this study is to investigate the possibility of establishing some suitable seismic coefficients for the design of industrial structures in various active earthquake zones in this country (See Fig. 1).

The mass of a structure, including equipment and live loads, plays an important part in a seismic design. Since it is basically a dynamics problem, the fundamental period—or rigidity—of a structure largely determines the magnitude of the induced seismic forces. Generally speaking, flexible structures having longer natural periods respond less than rigid structures. The distribution of seismic forces, such as base shear, is simplest with a concentrated mass (one-mass system), whereas in a uniform-cantilever structure (equivalent one-mass system), the distribution will vary to a greater extent, particularly for the higher modes of vibration.

The effects of both structural and hysteresis damping in the response of structures have been investigated to a certain extent.(3,18,19,23) The magnitude of the damping force is commonly expressed as a percentage of that critical value of damping at which the motion of the structure loses its vibratory character if subjected to a sudden impulse. Since there are numerous factors, such as joint friction, heat generation, and dislocation and plastic deformation of the ground and the structure itself, which contribute to the dissipation of energy, no general, reliable information concerning the magnitude of the damping in various structures has yet been established. After a review of the results obtained by recent investigators(3,15,19) a tentative value of 10 per cent of critical damping has been chosen for structures being considered in this study. The values yielded by the review were in the order of 3 to 6 per cent for steel frames and 7 to 14 per cent for concrete buildings, with the greater amounts for more brick walls and partitions.

Ground motion is the prime factor which generates the entire range of the dynamic responses of a structure. The characteristics of recorded ground movements vary greatly. However, it is interesting to note that maximum accelerations of major recorded shocks occur with almost identical periods. These range approximately from 0.1 to 0.5 sec.

The rigidity of the ground also affects the response of the structures to a certain extent. Since current knowledge is inadequate to establish numerical values for ground rigidity, it is assumed that the structures considered in this investigation are founded on firm soil.

Dynamic Analysis of Typical Structures

Response Spectra

The response spectrum, or the maximum shear influence line, may be defined as the curve which represents the maximum seismic base shear as a function of the undamped fundamental period of the single-mass structure. This so-called response spectrum technique was originally presented by Prof. M. A. Biot about 17 years ago.(1) Later, response spectra based on certain recorded shocks were evaluated by Professors J. L. Alford, G. W. Housner, D. E. Hudson, and others.(2,4,12,13,14) Unfortunately their method

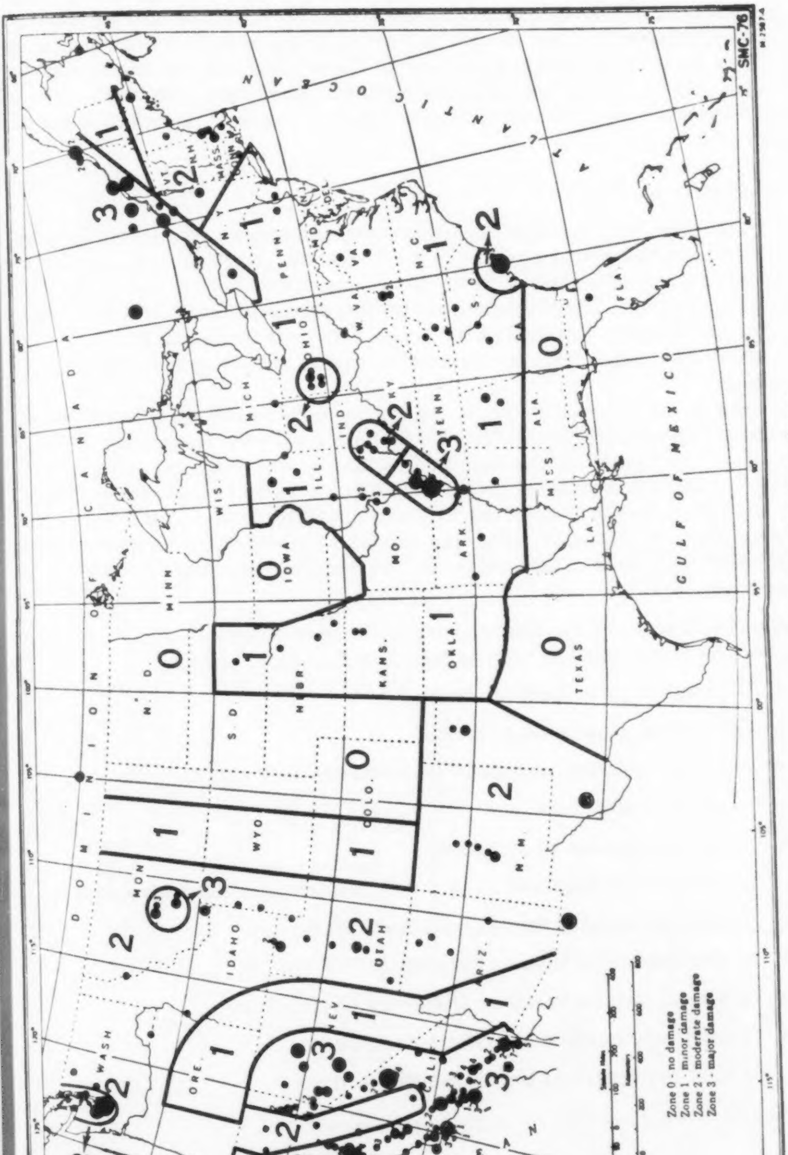


Fig. 1 Map of the United States Showing Zones of Approximately Equal Seismic Probability

(Reproduced from the uniform Building Code).

of determining the seismic forces induces in structures has as yet not been adopted in any building code.[6,7,8,10] It is true that the Joint Committee of ASCE and Structural Engineers Association of Northern California employed this technique to set forth a standard spectrum.[11] Their report is, however only a recommendation, and its use is limited to the design of ordinary buildings. This paper is an outgrowth of one of the suggestions for future study made by the joint committee. It is the author's intention to establish a set of standard response spectra which may be used in aseismic design of any kind of structure.

One-Mass System

Theoretically, the idealized one-mass system represents a structure with all the inertia loads acting at a single point. On the practical side, however, simple structures with most of their mass concentrated at one level may be put in this category. There are numerous structures of this kind: One-story buildings and frames, elevated water tanks, conveyor bents and towers, single-story pipe supports, and horizontal vessels and equipment supports, for example.

Selection of representative recorded earthquakes is essential to establish the response spectra of the one-mass system for various earthquake zones, (See Fig. 1). On reviewing the damages caused by the recorded major shocks[5,16] and the results of recent studies by engineering seismologists, 50 per cent (N-S Component) of the El Centro Accelerogram of the Imperial Valley earthquake of May 18, 1940 was chosen as the average intensity for Zone 3. The magnitude of this particular earthquake was about 6.7 on the Richter scale.[21]

Equations of Motion and the Corresponding Computer Circuit

An idealized one-mass system is shown in Fig. 2.

x = absolute movement of structure

g = absolute movement of ground

z = $x - g$ = relative movement of structure with respect to ground

m = mass of structure

k = spring constant of structure

= shear force required to give unit deflection in the z direction

c = viscous damping force coefficient, force = $c\dot{z}$

V = maximum base shear of structure

w_n = $\sqrt{k/m}$, natural circular frequency of structure

n = $c/2\sqrt{km}$, fraction of critical damping

$\dot{x}$ = first derivative of x with respect to time.

According to Newton's Law,

$$m\ddot{x} + c\dot{z} + kz = 0$$

$$\text{or} \quad m(\ddot{z} + \ddot{g}) + c\dot{z} + kz = 0$$

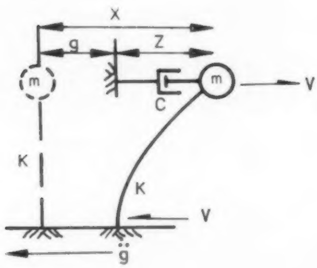


Fig. 2 One Mass System

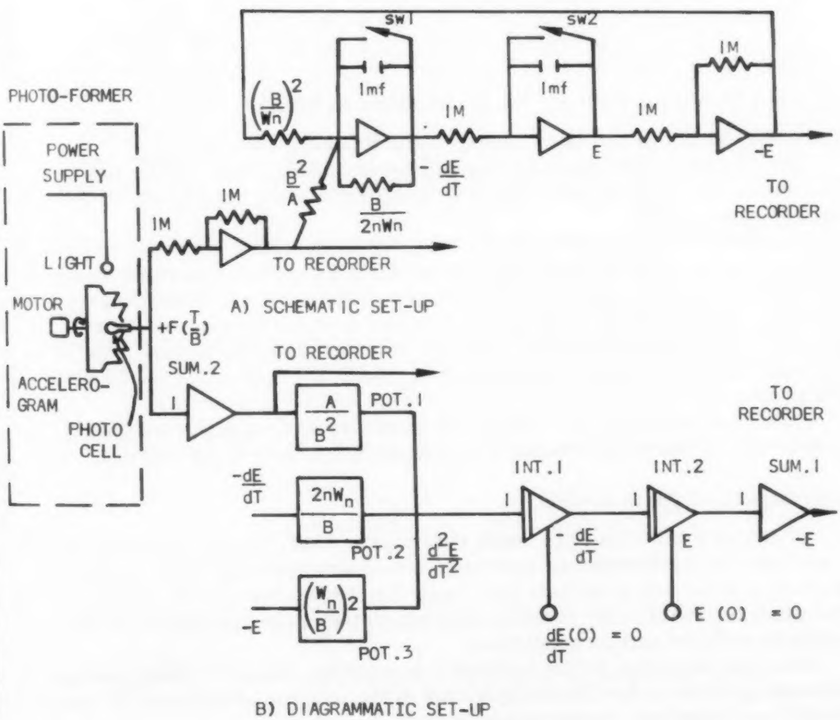


Fig. 3 Computer Circuit for Eq. (3).

$$\ddot{z} = -\ddot{g} - \frac{c}{m}\dot{z} - \frac{k}{m}z \tag{1}$$

$$\text{or} \quad -\dot{z} = \int \ddot{g}\, dt + 2nw_n \int \dot{z}\, dt + w_n^2 \int z dt \tag{2}$$

$$\text{Let } T = Bt, \quad E = Az \quad \text{and} \quad \ddot{g} = F(t)$$

Then

$$\frac{d}{dt} = B\frac{d}{dt}$$

$$\frac{d^2}{dt^2} = B^2 \frac{d^2}{dt^2}$$

$$z = \frac{E}{A} \quad \text{and} \quad \ddot{g} = F(\tfrac{t}{B})$$

After substituting the above expressions into Eq. (2), the following equation is obtained

$$-\frac{dE}{dT} = \frac{A}{B^2}\int F(\tfrac{T}{B})\, dT + 2nw_n \frac{1}{B}\int \frac{dE}{dT}\, dT + (\frac{w_n}{B})^2 \int E dT \tag{3}$$

The computer circuit for Eq. (3) is shown in Fig. 3.

Recorded Solutions and the Corresponding Response Spectra

The Berkeley Electronic Analog Computer (EASE Model 1032), Sanborn Recorder, and Photoformer Function Generators were used to obtain the response spectra.[20]

To gain a better understanding of the variations of the response in the system due to different damping factors, response spectra were established for n = 0.5, n = 0.10, and n = 0.20. The recorded solutions thus obtained are partially shown in Figs. 4, 5, and 6, together with the forcing function F(T/B). All results evaluated from the recorded data are shown in Table 1. The acceleration response spectra for n = 0.05, n = 0.10 and n = 0.20 are plotted in Fig. 7.

Undamped response (n = 0.0) of one mass system has also been investigated. Fig. 8 shows the response variations for different structures.

Equivalent One-Mass System

All complex structures in which the mass is concentrated on two or more floors may be considered as equivalent one-mass systems. In practice, the mass of a structure is seldom uniformly distributed over all its levels. In this study, however, only structures in which the mass distribution is essentially uniform will be considered.

Since the response, or the maximum base shear, due to a given earthquake depends upon the deflected configuration of the system, structures of this kind should be divided into three categories:

(1) Shear-deflection type
(2) Moment-deflection type
(3) Intermediate type

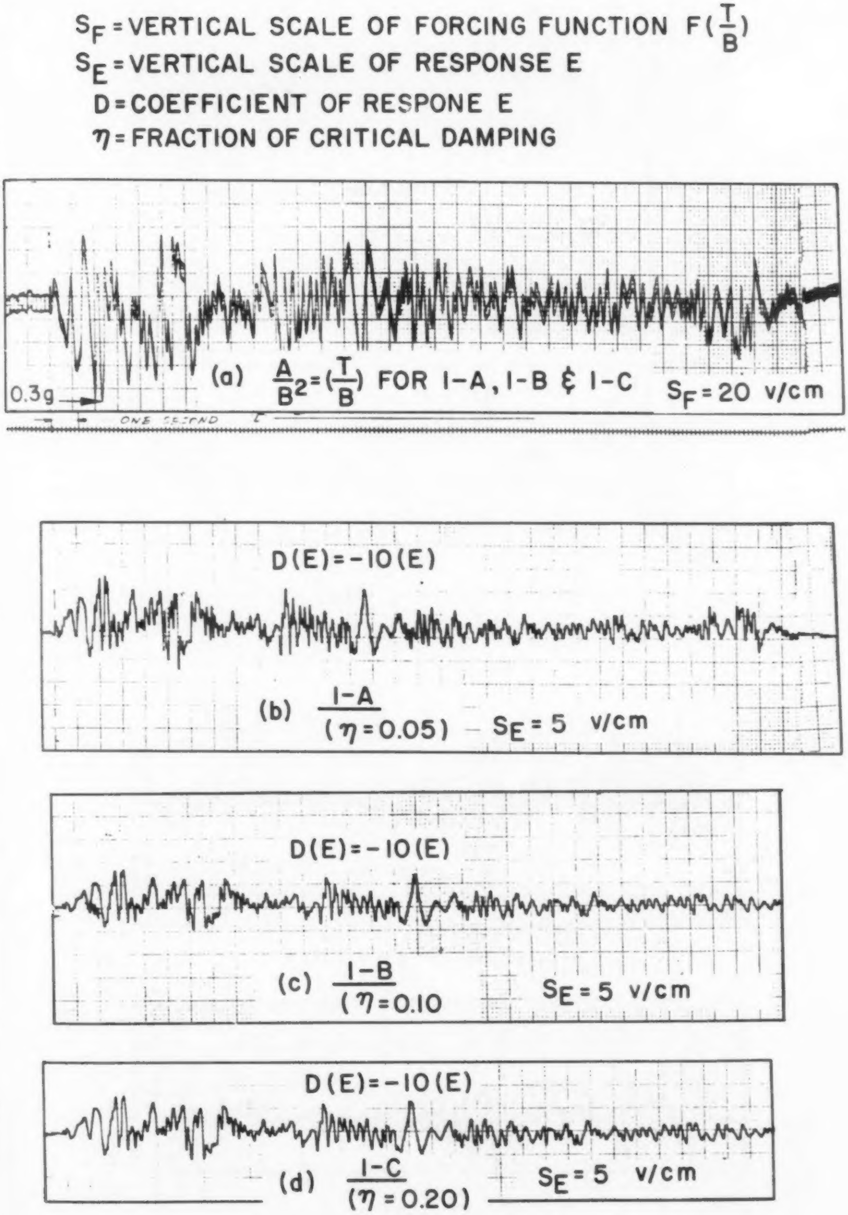


Fig. 4 Recorded Response for 1-A, 1-B, and 1-C (T = 0.1 sec)

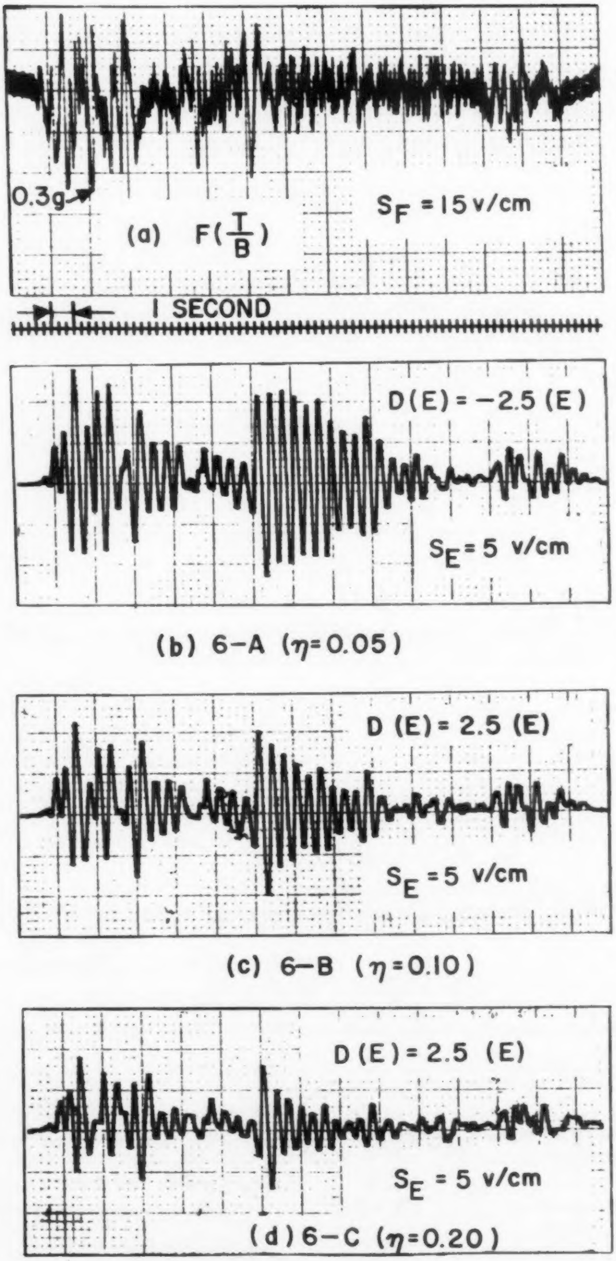


Fig. 5 Recorded Response for 6-A, 6-B, and 6-C (T = 0.6 sec)

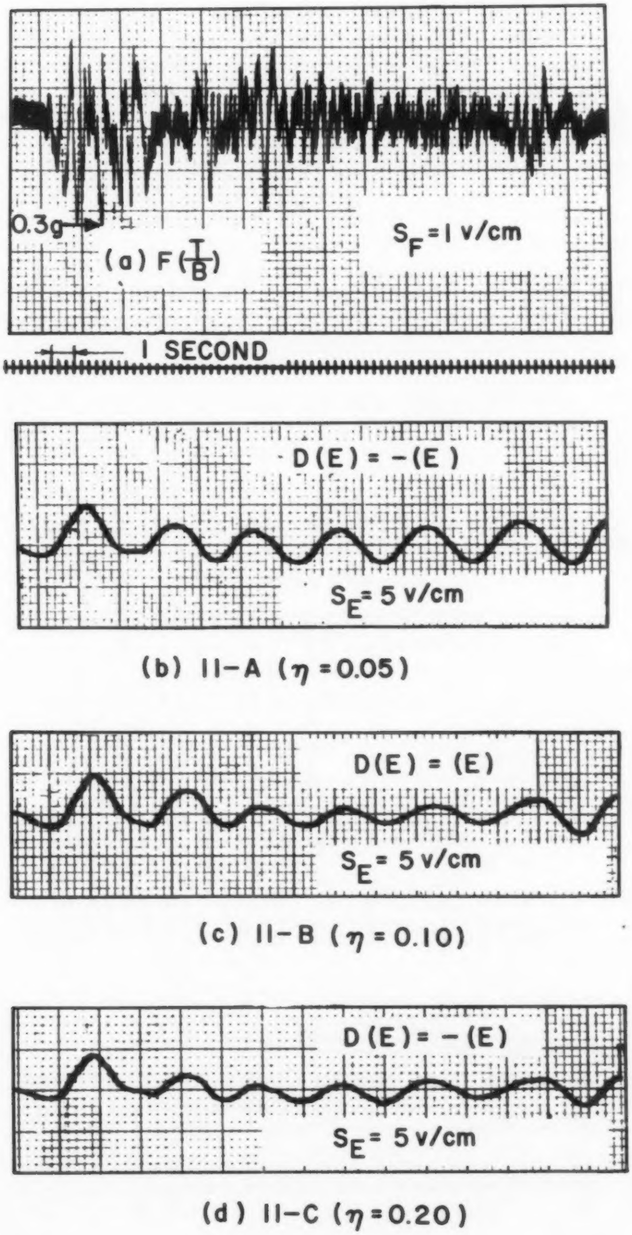


Fig. 6 Recorded Response for 11-A, 11-B, and 11-6 (T = 4.0 sec)

TABLE 1 - BASIC RESPONSE DATA FOR ONE-MASS SYSTEM

ITEM	FUND. PERIOD T (sec)	FREQUENCY $f=\frac{1}{T}$ (cps)	FREQUENCY $W_n=\frac{2\pi}{T}$ (rps)	SPRING CONSTANT $k=mw_n^2$ (m=1)	B	$A=B^2$	$(\frac{W_n}{B})^2$	$\frac{2\eta W_n}{B}$	DAMPING FACTOR η	* MAX. E x (10^{-2})	MAX. Z=E/A(10^{-2})	MAX. BASE SHEAR V=KZ	REFERENCE
1-A	0.1	10.00	62.8	3948.0	6.0	36.0	109.5	1.04	0.05	0.32	0.009	0.35	Fig.4
1-B	0.1	10.00	62.8	3948.0	6.0	36.0	109.5	2.08	0.10	0.26	0.007	0.28	
1-C	0.1	10.00	62.8	3948.0	6.0	36.0	109.5	4.16	0.20	0.21	0.006	0.24	
2-A	0.2	5.00	31.4	986.0	3.0	9.0	109.5	1.05	0.05	0.70	0.078	0.77	
2-B	0.2	5.00	31.4	986.0	3.0	9.0	109.5	2.10	0.10	0.45	0.050	0.49	
2-C	0.2	5.00	31.4	986.0	6.0	36.0	27.4	2.10	0.20	1.08	0.030	0.29	
3-A	0.3	3.33	20.9	439.0	2.5	6.25	70.0	0.84	0.05	1.27	0.203	0.90	
3-B	0.3	3.33	20.9	439.0	2.5	6.25	70.0	1.68	0.10	0.85	0.135	0.60	
3-C	0.3	3.33	20.9	439.0	2.5	6.25	70.0	3.36	0.20	0.59	0.094	0.42	
4-A	0.4	2.50	15.7	246.0	2.5	6.25	39.3	0.62	0.05	13.6	2.190	0.86	
4-B	0.4	2.50	15.7	246.0	2.5	6.25	39.3	1.24	0.10	10.4	1.670	0.67	
4-C	0.4	2.50	15.7	246.0	2.5	6.25	39.3	2.48	0.20	8.55	1.370	0.54	
5-A	0.5	2.00	12.6	158.0	2.5	6.25	25.2	0.50	0.05	21.4	3.42	0.86	
5-B	0.5	2.00	12.6	158.0	2.5	6.25	25.2	1.00	0.10	15.9	2.54	0.64	
5-C	0.5	2.00	12.6	158.0	2.5	6.25	25.2	2.00	0.20	11.7	1.87	0.47	
6-A	0.6	1.67	10.5	109.0	2.5	6.25	17.4	0.42	0.05	25.4	4.08	0.71	Fig.5
6-B	0.6	1.67	10.5	109.0	2.5	6.25	17.4	0.84	0.10	19.0	3.04	0.53	
6-C	0.6	1.67	10.5	109.0	2.5	6.25	17.4	1.68	0.20	14.7	2.35	0.41	
7-A	0.7	1.43	9.0	81.0	2.5	6.25	13.0	0.36	0.05	28.6	4.60	0.60	
7-B	0.7	1.43	9.0	81.0	2.5	6.25	13.0	0.72	0.10	21.5	3.46	0.45	
7-C	0.7	1.43	9.0	81.0	2.5	6.25	13.0	1.44	0.20	16.2	2.55	0.34	
8-A	0.8	1.25	7.9	61.6	2.5	6.25	9.8	0.31	0.05	31.9	5.10	0.52	
8-B	0.8	1.25	7.9	61.6	2.5	6.25	9.8	0.62	0.10	24.5	3.92	0.40	
8-C	0.8	1.25	7.9	61.6	2.5	6.25	9.8	1.24	0.20	16.0	2.55	0.26	
9-A	1.0	1.00	6.3	39.4	2.5	6.25	6.3	0.25	0.05	41.7	6.67	0.42	
9-B	1.0	1.00	6.3	39.4	2.5	6.25	6.3	0.50	0.10	33.8	5.40	0.34	
9-C	1.0	1.00	6.3	39.4	2.5	6.25	6.3	1.00	0.20	20.2	3.22	0.20	
10-A	2.0	0.50	3.1	10.0	2.5	6.25	1.6	0.12	0.05	66.5	10.65	0.17	
10-B	2.0	0.50	3.1	10.0	2.5	6.25	1.6	0.24	0.10	55.0	8.80	0.14	
10-C	2.0	0.50	3.1	10.0	2.5	6.25	1.6	0.48	0.20	47.0	7.50	0.12	
11-A	4.0	0.25	1.6	2.5	2.5	6.25	0.4	0.06	0.05	203.0	33.5	0.13	Fig.6
11-B	4.0	0.25	1.6	2.5	2.5	6.25	0.4	0.12	0.10	188.0	31.0	0.12	
11-C	4.0	0.25	1.6	2.5	2.5	6.25	0.4	0.24	0.20	171.0	28.4	0.11	

* Maximum values of "E" are computed from the recorded maximum response of the system

Typical shear-deflection structures are multi-story buildings with hinged joints and cross-bracings, with or without masonry walls (mill, office and service buildings; power, boiler and cast houses, docks, and warehouses).

Typical moment-deflection structures are multi-story buildings with moment connections, chimneys, slender vertical vessels, stand pipes, blast furnaces and the like.

Intermediate structures are combinations of the other two types, including the pedestals supporting heavy equipment (motor- and turbo-generators, compressors, kilns, bins, and storage tanks).

The system under consideration has many degrees of freedom, and its structures will vibrate in different modes. However, the analysis of such a system may be simplified by studying the fundamental mode only, which will provide the maximum base shear. Although maximum stresses at higher levels of the structure are usually induced by higher modes, these stresses may be taken care of by a proper distribution of the maximum base shear obtained from the fundamental mode.(11)

Responses of undamped two-mass and three-mass structures respectively have been obtained by means of the analog computer. Figs. 9 and 10 show the results of the structures analyzed.

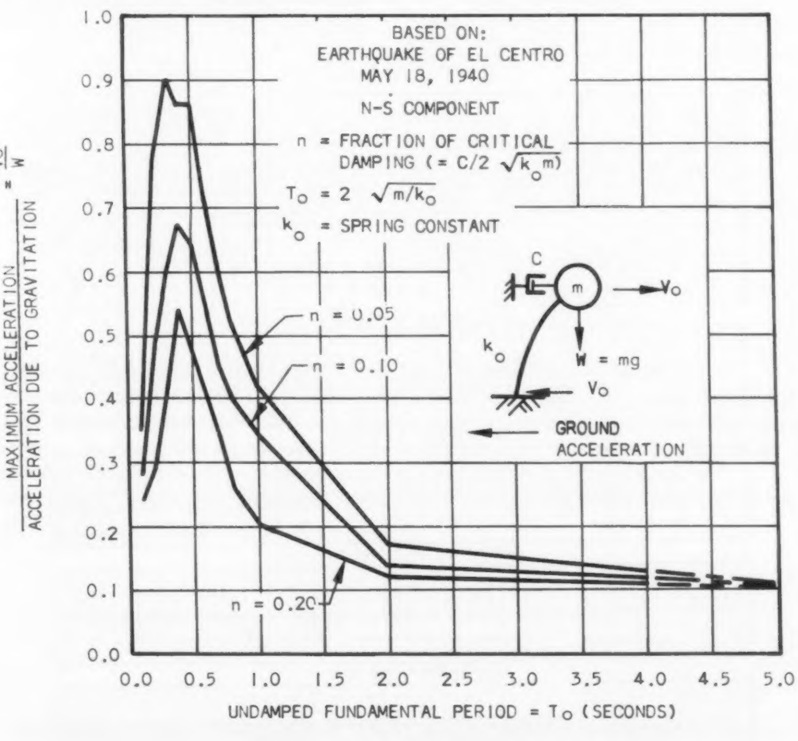


Fig. 7 Acceleration Spectra for Damped One-Mass System

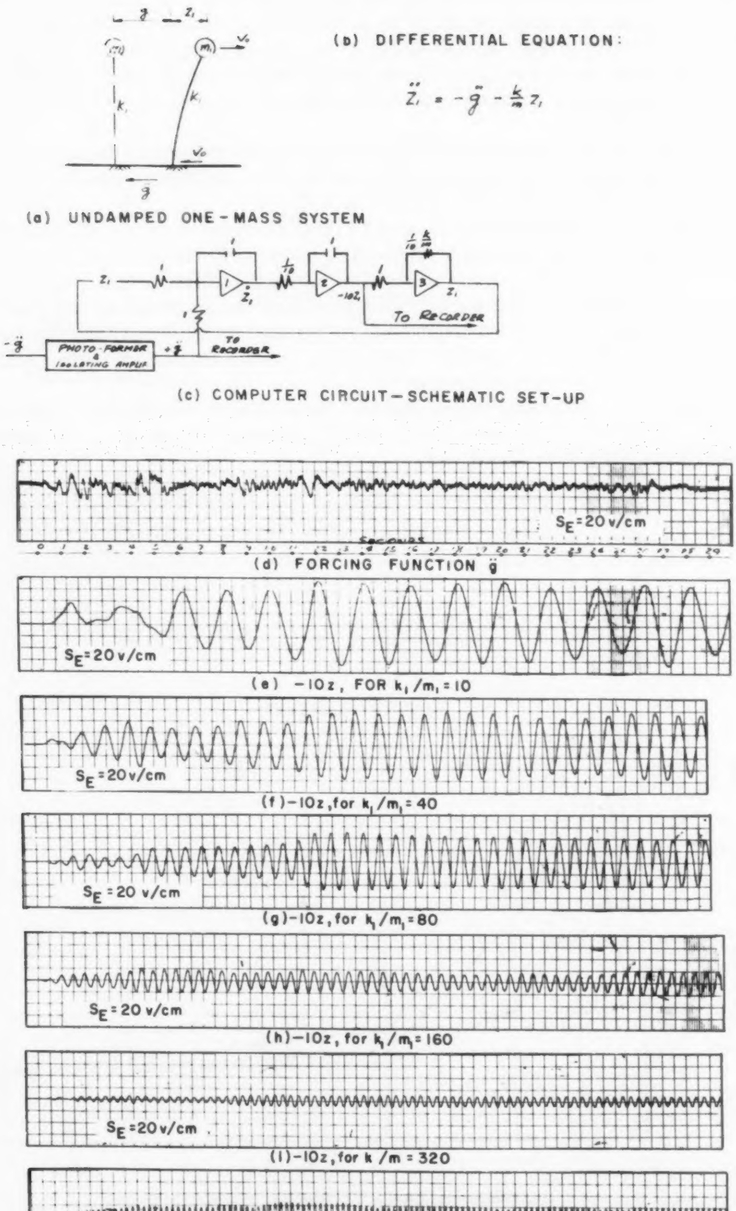


Fig. 8 Recorded Response of Undamped One-Mass System

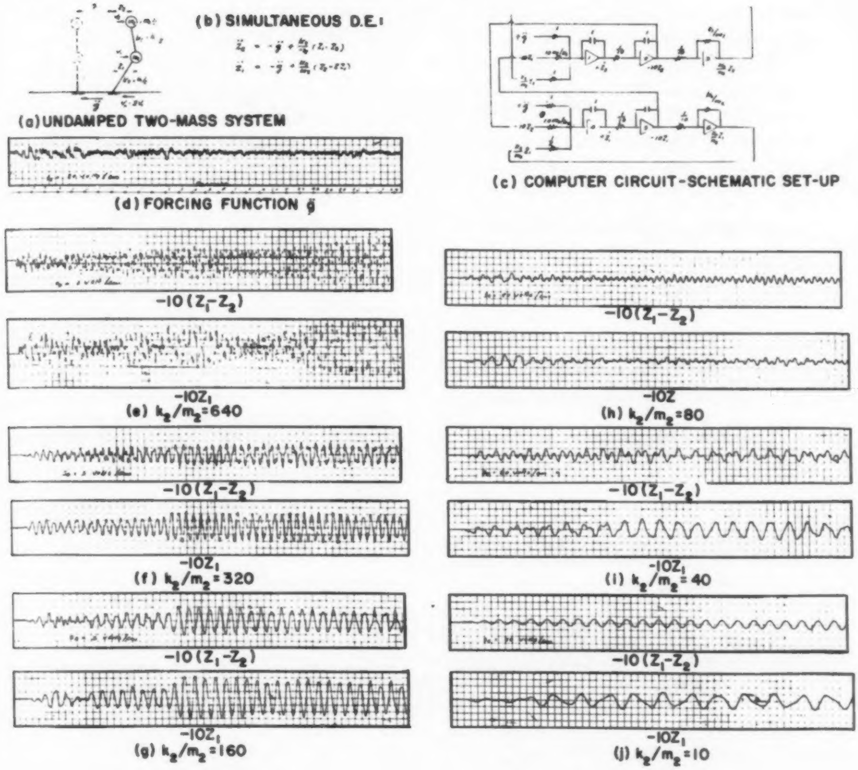


Fig. 9 Recorded Response of Undamped Two-Mass System

Although response spectra for these limited-degrees-of-freedom structures can be established, a general spectrum which will furnish the particular seismic coefficient for any degree-of-freedom or equivalent one-mass structure is even more practical. The basic method of evaluating the base shear for the equivalent mass system was established by Prof. M. P. White.(26) The following general formula is readily obtained from his work (See Fig. 11):

$$V_e = \frac{1}{L} \frac{\left[\int_0^L z(y)\, dy\right]^2}{\int_0^L z^2(y)\, dy} V_o \tag{4}$$

The shape of the shear-deflection of the structure may be approximated from the curve $Z(y) = Z_1 \cos \frac{y}{2L}$ for the fundamental mode of vibration (Z_1 denotes the maximum deflection). Similarly one substitutes $Z(y) = Z_1 (\frac{y}{L})^2$ for the moment deflection to the structure.

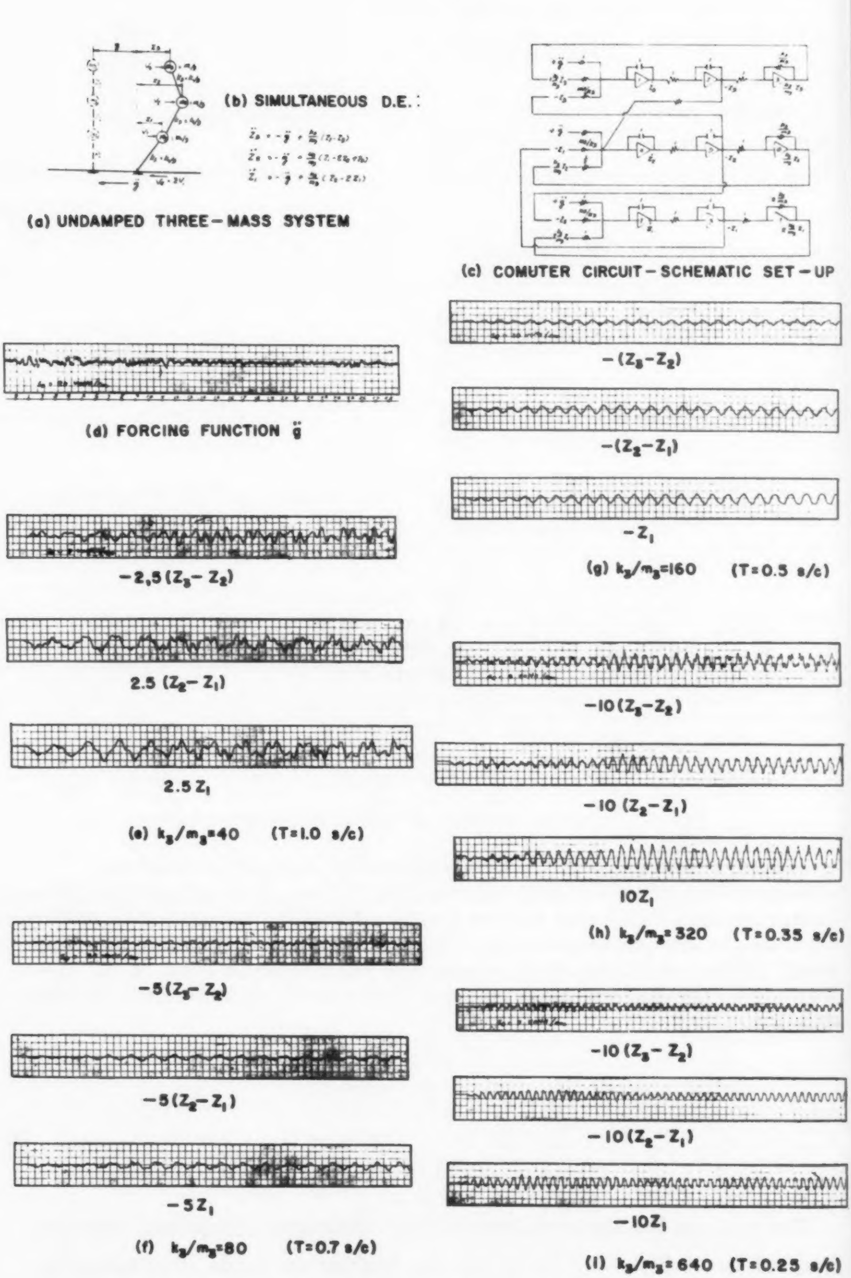
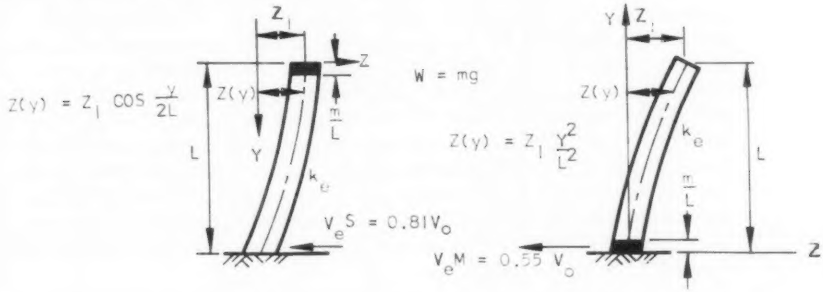


Fig. 10 Recorded Response of Undamped Three-Mass System

The spectral values of the system arising from combinations of shear and moment deflections can be approximated similarly by assuming a reasonable mode shape of the structure. However, since the actual percentages of deflections induced by shears and moments cannot be predicted, and since an actual multiple-story structure will vary from the idealized structure, an average value of the shear add moment-deflection cases may be chosen as a representative value for this system.



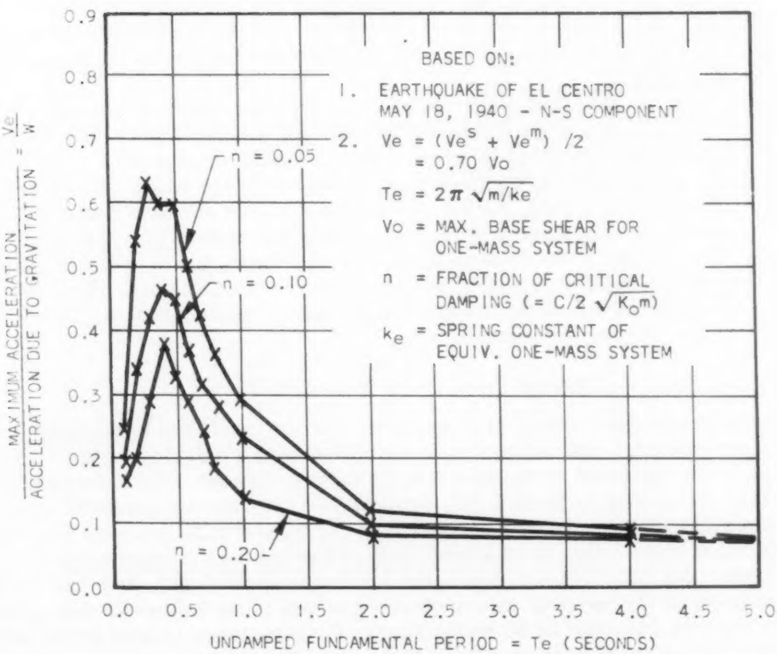


Fig. 11 Acceleration Spectra for Damped Equivalent One-Mass System

Accordingly, the response spectra for the equivalent one-mass system is established as shown in Fig. 11.

Interpretation of Analysis

The following general conclusions may be drawn:

(1) For a given earthquake, the maximum base shear of a structure varies according to its rigidity or fundamental period. If a damping of n = 0.10 is selected as the standard for all structures, then the absolute maximum base shear amounts to 0.67W for the one-mass system, and 0.47W for the equivalent one-mass system; these maximum values are generated in structures with a fundamental period of T = 0.45 sec. It would be noted that these results were obtained through use of a forcing function equal to the maximum ground acceleration recorded so far.

(2) The critical range of the fundamental periods of structures within which maximum base shears occur is approximately from 0.2 to 0.5 sec. per cycle. Beyond both limits, the response diminishes and theoretically drops to 0 at T = 0 and T = ∞.

(3) It may be noted in Figs. 4, 5, and 6 that the maximum deflections of damped structures immediately follow the maximum strokes in the forcing function. In other words, the same results, approximately, could have been obtained if only the first few seconds of the forcing function had been introduced. However, in case of undamped structures, the response continues to build up as long as the ground is in motion. (See Fig. 8)

(4) From the recorded response of two- and three-story structures, (Figs. 9 and 10), it may be observed that for uniform structures, the induced transient vibration is dominated by the fundamental mode.

(5) If a given mass is distributed uniformly along the height of a structure of constant rigidity, the induced maximum base shear is reduced. The amount of reduction is greater for a moment-deflection type structure than that for a shear-deflection type. Assuming the deflection is 50 per cent due to bending and 50 per cent due to shear, the base shear for this equivalent one-mass system is equal to approximately 70 per cent of that for the one-mass system.

Recommendation of Design Seismic Coefficients(24)

From these conclusions, it is apparent that the maximum base shear is too high, for practical and economical design even with a damping factor n = 0.10. This can be explained by the fact that the forcing function selected was too severe. According to Prof. C. W. Housner,(22) the velocity spectrum intensity of this particular earthquake is about 2.71 fps for n = 0.20, whereas the corresponding average value of 14 strong-motion earthquakes is approximately 1.31 fps. The ratio of the latter to the former is about 0.483. In other words, 50 per cent of the spectral values obtained in this study represent the approximate average response of 14 major shocks for structures located in the active earthquake zones in this country.

The response spectra have been modified accordingly. Fig. 12 shows the final shape of the spectrum for Zone 3, and the adjusted curve assumes the proposed design seismic coefficient spectrums for a one-mass system. The

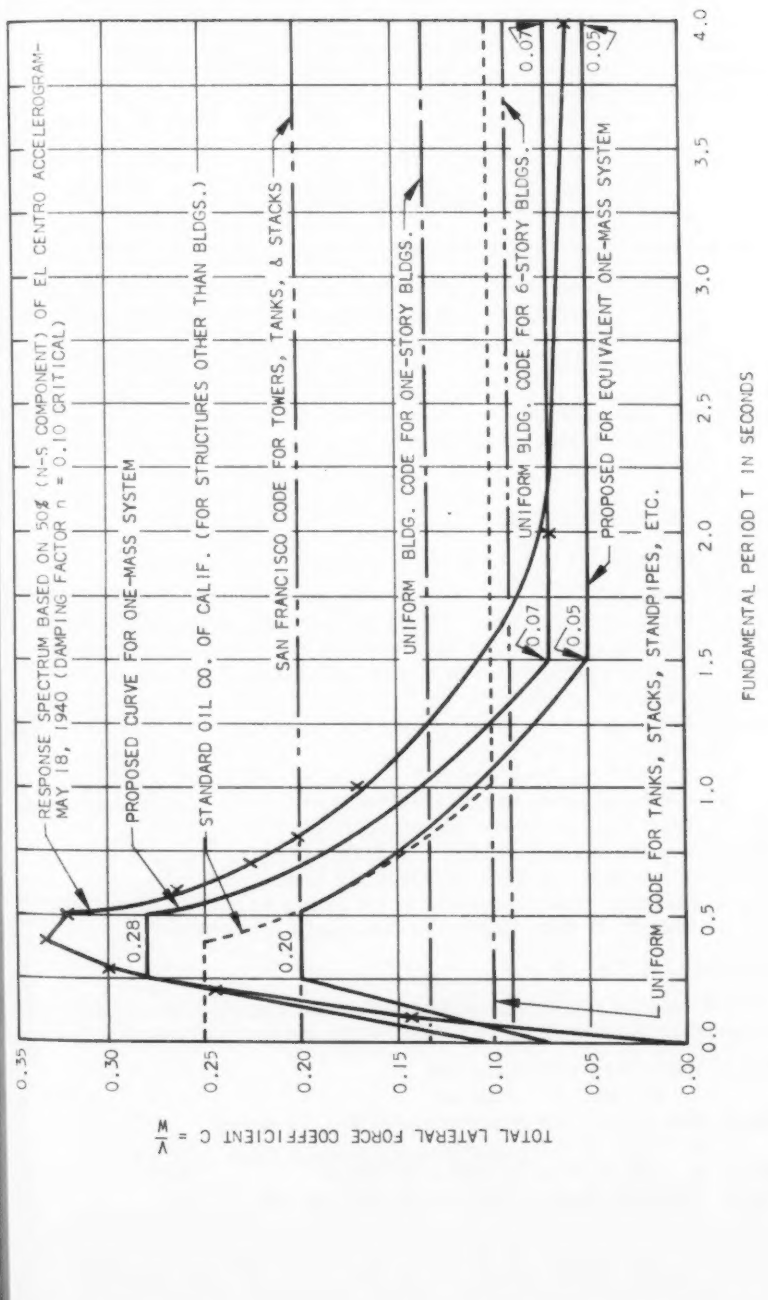


Fig. 12 Proposed Design Seismic Coefficients for Zone 3

The standard spectrum for the equivalent one-mass system is also plotted, based on the reasoning used in Point 5 of the general conclusions.

The response spectra for Zones 2 and 1 are also established following the same concept as that used in the Uniform Building Code.[7] A c-value for Zone 2 and Zone 1 is equal to 0.5 and 0.25, respectively, of that for Zone 3. Figs. 13 and 14 show the final form of the proposed curves.

For comparison, values of seismic coefficients for various structures given by recognized authorities are also plotted in Fig. 12.

APPENDIX A

Illustrative Examples

The design procedure for any structure consists of four steps:[25]

Step 1 Determination of the fundamental period, T. Formulas for determining the period of structures are given in Appendix B.

Step 2 Determination of the lateral force coefficient, c.

Step 3 Computation of the total lateral shear applied to the structure, $V = cw$.

Step 4 Distribution of this total shear along the height of the structure. The loading thus obtained is to be used in the static analysis of the structure.

Design of One-Mass Structures

(1) Structures A and B in Fig. 15

Step 1 Using the method of virtual work, the total elastic deformation (Δ) at level AB due to inertia load acting horizontally is 0.829 in. and 39,000 in. for Structures A and B respectively, the corresponding exact period is 0.29 and 2.00 sec. as shown in Table A-1.

Step 2 For earthquake Zone 3, using the upper curve in Fig. 12 $c = 0.25$ for Structure A
$c = 0.20$ for Structure B
(Corresponding values for Zone 2 or Zone 1 may be determined from Fig. 13 or Fig. 14, respectively.)

Step 3 $V = cW = 0.25 \times 100{,}000 = 25{,}000$ lb for Structure A
$V = 0.10 \times 375{,}000 = 37{,}500$ lb for Structure B

Step 4 The total shear is applied at the center of gravity of the inertia load W, at floor line AB of Structures A and B.

(2) Building C in Fig. 15

Lateral seismic force may be determined for the preliminary design of a one-story building of given height, width and inertia load as follows:

Step 1 Here $H^2/b = 63^2/20 = 200$
By Eq. B-2, $T = 1.00$ sec.

Step 2 For Zone 3, the upper curve of Fig. 12 shows
$c = 0.15$ for $T = 1.11$ sec.

Step 3 $V = cW = 0.15 \times 100{,}000 = 15{,}000$ lb.

Step 4 The total shear is applied at roof line AB.

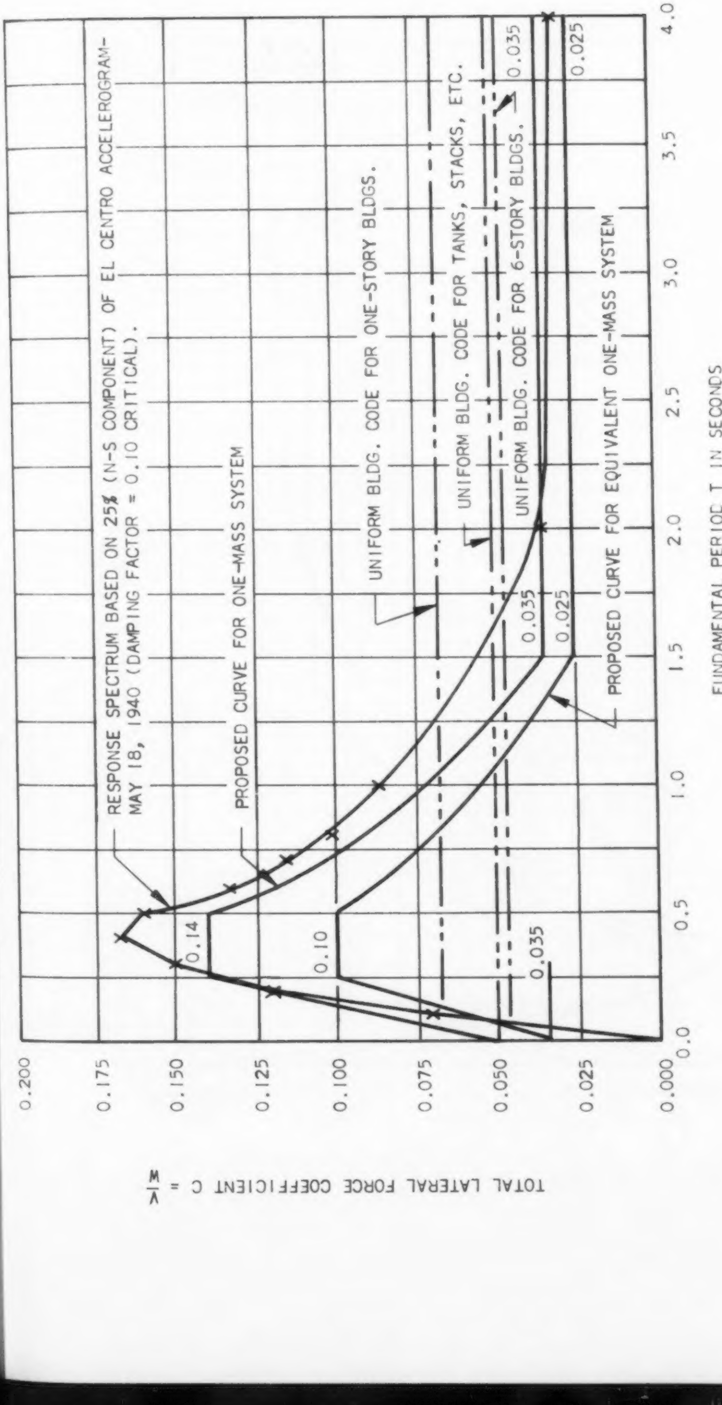


Fig. 13 Proposed Design Seismic Coefficients for Zone 2

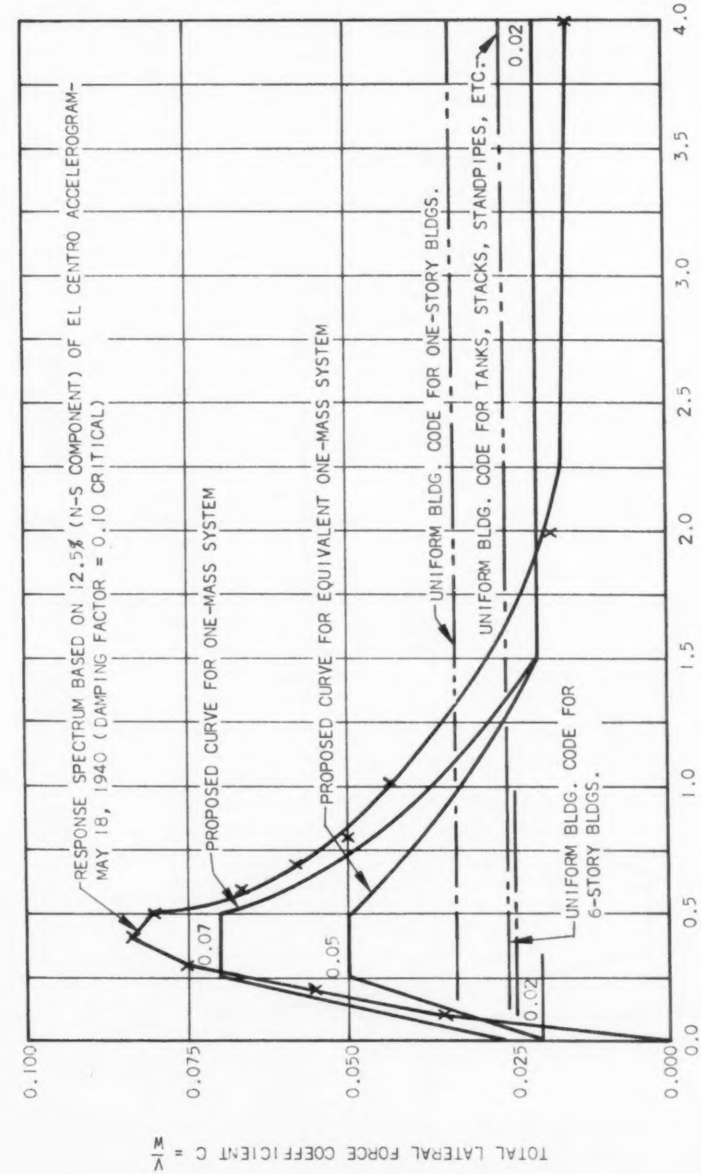


Fig. 14 Proposed Design Seismic Coefficient for Zone 1

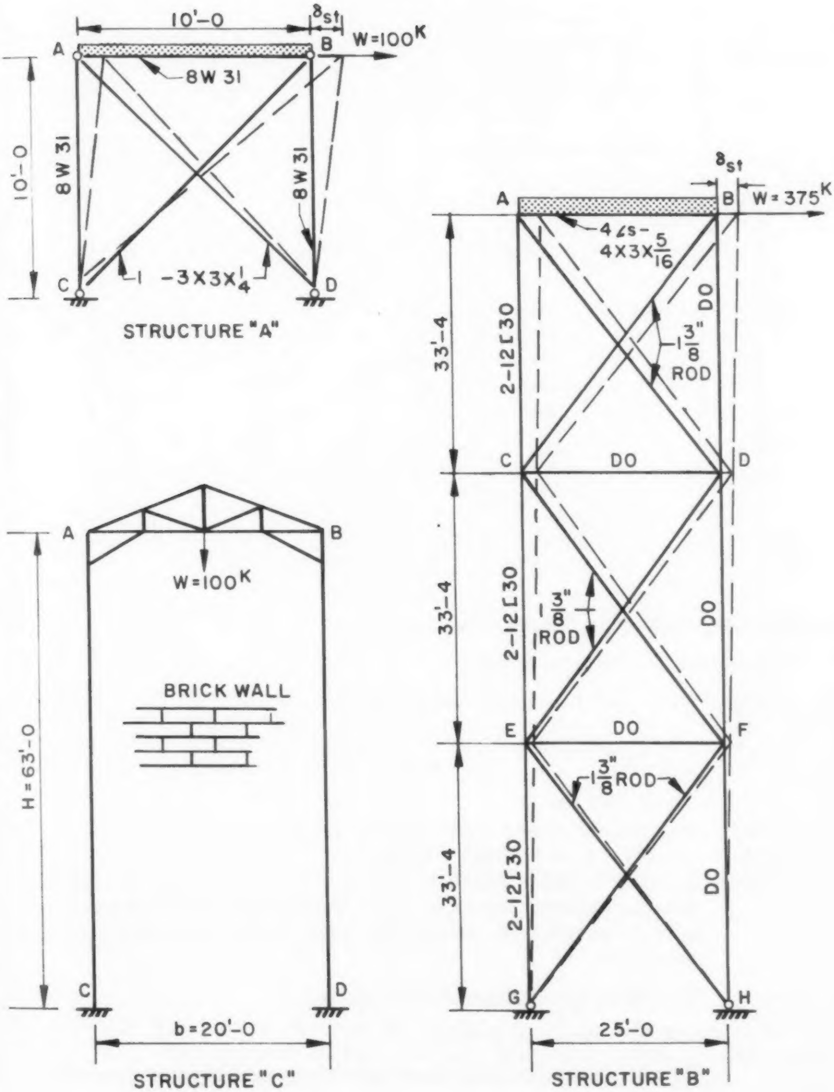


Fig. 15 One-Mass Structures

Table A-1 COMPUTATION OF PERIOD FOR STRUCTURES A and B

STRUCTURE	MEMBER	LENGTH, L (in.)	STRESS, S (lb.)	AREA, A (in.^{2})	$\frac{SL}{AE}$	u	$\frac{SL}{AE}u$	PERIOD, T (SEC)
A	AB	120	0	9.12	0	0	0	$T_{exact} = 2\pi\sqrt{\frac{0.829}{32.2\times12}}$
	AD	170	0	1.44	0	0	0	$= 0.29$ sec
	AC	120	0	9.12	0	0	0	
	BD	120	-100,000	9.12	-0.044	-1.00	+0.044	$T_{approx} = 2\pi\sqrt{\frac{0.785}{32.2\times12}}$
	BD	170	+141,400	1.44	+0.557	+1.41	+0.785	$= 0.28$ sec
	Exact $\delta_{st} = \Sigma SL\ u/AE = 0.829$ in.							
B	AB	300	0	8.36	0	0	0	
	AC	360	0	17.58	0	0	0	$T_{exact} = 2\pi\sqrt{\frac{39.0}{32.2\times12}}$
	BC	470	-590,000	1.49	-6.20	-1.58	9.80	$= 2.0$ sec
	BD	360	+447,000	17.58	+0.30	+1.19	0.37	
	CD	300	+375,000	8.36	+0.45	+1.00	0.45	
	CE	360	-447,000	17.58	-0.30	-1.19	0.37	$T_{approx} = 2\pi\sqrt{\frac{29.4}{32.2\times12}}$
	DE	470	-590,000	1.49	-6.20	-1.58	9.80	$= 1.73$ sec
	DF	360	+894,000	17.58	+0.60	+2.38	1.43	
	EF	300	+375,000	8.36	+0.45	+1.00	0.45	
	EG	360	-894,000	17.58	-0.60	-2.38	1.43	
	FG	470	-590,000	1.49	-6.20	-1.58	9.80	
	FH	360	+1,341,000	17.58	+0.90	+3.58	3.22	
	Exact $\delta_{st} = \Sigma SL\ u/AE = 39.00$ in.							

NOTES: 1. $E = 30 \times 10^6$ lb/in^2 (steel)
2. Bracings assumed to take tension only
3. $T_{approx.}$ = Approximate period computed based on the approx. δ_{st} caused by the strains in the diagonal members only.

Design of Equivalent One-Mass Structures

(1) Structure D—Uniform Stack

Step 1 Referring to Fig. 16 and Eq. B-3 of Appendix B

$$T = 1.785(100 \times 12)^2 \sqrt{\frac{0.0788}{30 \times 10^6 \times 107 \times 10^3}}$$

$$= 0.401 \text{ sec.}$$

Step 2 For Zone 3, lower curve of Fig. 12 gives c = 0.18

Step 3 V = cW = 0.18 x 36500 = 6.600 lb.

Step 4 In order to take care of the maximum stresses due to higher modes properly, the total shear should be distributed along the height triangularly. Hence, the shear at the top of the structure should be

$V_t = 6600/\frac{L}{2} = ;3200/100 = 132$ lb/ft.,

and V_b = shear at bottom = 0

Fig. 16 shows the final shear distribution along the structure.

(2) Structure E—Flared Stack

Step 1 By conjugate beam method, the structural deflection due to inertial loads acting horizontally is obtained as shown in Fig. 16B. Using Eq. B-4 of Appendix B, the period T can be evaluated as follows:

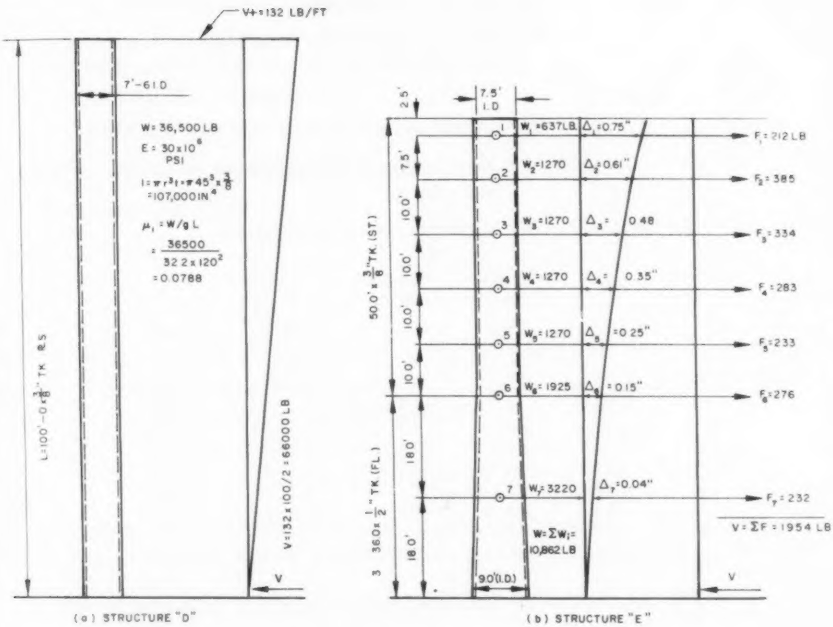


Fig. 16 Equivalent One-Mass Structures

$$\sum_{i=1}^{i=7} W_i \Delta_i = 3043$$

and

$$\sum_{i=1}^{i=7} W_i \Delta_i = 1408$$

$$T = 0.32 \sqrt{\frac{1408}{3043}} = 0.22 \text{ sec.}$$

Step 2 **The lower curve of Fig. 12 gives c = 0.18 for Zone 3.**

Step 3 $W = \Sigma W_i = 10862$ lb

$V = cW = 0.18 \times 10862 = 1{,}966$ lb

Step 4 **For the structure under consideration and other similar multistory structures, the total lateral force V should be distributed over the height of the structure in accordance with the following formula:(11)**

$$F_i = V \frac{W_i h_i}{\Sigma W_i h_i}$$

Where F_i = lateral force applied to any level i

W_i = the vertical load, dead load plus percentage of live load, if any, at any tributary to the level i

h_i = height in feet of level i above the base

$\Sigma W_i h_i$ = summation of the products of all $W_i h_i$ for the structure.

Accordingly, the following result is obtained:

Level i	Load W_k (lb)	Height H_i (ft)	Product $W_i h_i$ (lb-ft)	Fraction $\frac{W_i h_i}{\Sigma W_i h_i}$	F_i (lb)
7	3220	18	57,960	0.1182	232
6	1925	36	69,300	0.1413	276
5	1270	46	58,420	0.1191	233
4	1270	56	71,120	0.1452	283
3	1270	66	83,820	0.1709	334
2	1270	76	96,520	0.1968	385
1	637	83.5	53,190	0.1085	212
	10862		490,330	1.0000	1955

Fig. 16 shows the final shear distribution at different levels of the structure.

APPENDIX B

Determination of Period

A few simple formulas are compiled below for evaluating fundamental periods of structures.

For one-story buildings with or without ordinary light siding and other structures:

$$T = 0.32\sqrt{\Delta} \qquad \text{B-1}$$

where T = undamped fundamental period in seconds

Δ = horizontal deflection in inches at the centroid of total inertia load acting horizontally

For buildings with masonry walls, approximate values may be had by the following statistical approach originated by the Joint Committee*

$$T = 0.07\ H/\sqrt{b} \qquad \text{B-2}$$

Where H = height of building

b = width of building in the direction considered

*Joint Committee(17) recommends $T = 0.05H/\sqrt{b}$, which seems too conservative when applied to the curves proposed in this paper.

For uniform cantilever structures

$$T = 1.785\ L^2\sqrt{\frac{\mu l}{EI}} \qquad \text{B-3}$$

where L = height of structure

μ = mass per unit height

EI = flexural rigidity or structure

For any building or structure

$$T = 0.32\sqrt{\frac{\Sigma W_i \Delta_i^{\,2}}{\Sigma W_i \Delta_i}} \qquad \text{B-4}$$

where ΣW_i = summation of distributed inertia load at i^{th} level above the base

Δ_i = horizontal deflection in inches at i^{th} level due to system of loads ΣW_i acting horizontally.

BIBLIOGRAPHY

1. "A Mechanical Analyzer for The Prediction of Earthquake Stresses," by M. A. Biot, Bulletin of The Seismological Society of America, Vol. 31, No. 2, April, 1941, p. 151.

2. "A Method of Calculating The Dynamic Force On A Building During An Earthquake," by Merit P. White, ibid; Vol. 32, No. 3, July, 1942, p. 193.

3. "Friction in Buildings: Its Magnitude and Its Importance In Limiting Earthquake Stresses," by Merit P. White, ibid; Vol. 31, No. 2, April, 1941, p. 93.

4. "Characteristics of Strong-Motion Earthquakes," by George W. Housner, ibid; Vol. 37, No. 1, January, 1947, p. 19.

5. "United States Earthquakes 1928-1954," by Frank Neumann, United States Coast and Geodetic Survey.

6. "Building Codes, City and County of San Francisco," Dept. of Public Works, San Francisco, Calif.; 1948.

7. "Uniform Building Code," Pacific Coast Building Officials Conference, Los Angeles, Calif.; 1955.

8. "Minimum Design Loads in Buildings and Other Structures," American Standards Association, New York, N. Y.; 1945.

9. "Building Code Requirements for Minimum Design Loads," ibid,; 1945.

10. "Recommendations-Earthquake Resistant Design of Buildings, Structures, and Tank Towers," Pacific Fire Rating Bureau, San Francisco, Calif.; 1950.

11. "Lateral Forces of Earthquake and Wind," by a Joint Committee of the San Francisco, California Section, ASCE, and the Structural Engineers Association of Northern California, Proceedings, ASCE, Vol. 77, Separate No. 66, 1951.

12. "Earthquake Resistant Design Based On Dynamic Properties of Earthquakes," by G. V. Housner, ibid; Vol. 28, No. 1, July, 1956.

13. "Structural Analysis by Dynamic Load Parameters," by J. A. Cheney, ibid; Vol. 28, No. 1, July, 1956.

14. "Spectrum Analysis of Strong-Motion Earthquakes," by G. W. Housner and J. L. Alford, Bulletin, Seismological Society of America, Vol. 43, No. 2, April, 1953.

15. "Dynamic Test of A Four-Story Reinforced Concrete Building," ibid; Vol. 43, No. 1, Jan., 1953.

16. "Seismicity of The Western United States," by P. Byerly, Proceedings of the World Conference On Earthquake Engineering, Earthquake Engineering Research Institute, San Francisco, Calif., 1956, p.1-1.

17. "Response Spectrum Techniques In Engineering Seismology," by D. E. Hudson, ibid; p. 4-1.

18. "Studies On The Nonlinear Vibrations of Structures Subjected To Destructive Earthquakes," by R. Tanabashi, ibid; p. 6-1.

19. "Vibrations of Buildings In Japan, Parts I & II," by H. Kawasumi, K. Kanai, T. Hisada and K. Nakagawa, ibid; p. 7-1.

20. "Investigation of The Utility of An Electronic Analog Computer In Engineering Problems," by D. W. Hagelbarger, C. E. Howe, and R. M. Howe, UMM-28, Dept. of Aeronautical Engineering, University of Michigan, Ann Arbor, Mich., 1949.

21. "An Instrumental Earthquake Magnitude Scale," by C. F. Richter, Bull. Seism. Soc. Am.; Vol. 25, p. 1, 1935.

22. "Symposium On Earthquake and Blast Effects On Structures," Earthquake Engineering Research Institute, San Francisco, Calif., 1952.

23. "Damping Capacity of Materials," by A. Yorgiadis Product Engineering, Nov., 1954.

24. "Proposed Design Seismic Coefficients for Industrial Structures," by Edward Y. W. Tsui, Kaiser Engineers, Oakland, Calif., 1957.

25. "Lateral Force Building Code Requirements", Recommendations of The Structural Engineers Association of Northern California, 1956.

26. M. P. White's Discussion of "Analytical and Experimental Methods In Engineering Seismology," by M. A. Biot, Transactions ASCE, Vol. 108, 1943.

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SELF-CHECKING GENERAL ANALYSIS OF RIGID FRAMES WITH SWAY

F. G. Keller[1]

SYNOPSIS

The method of Analysis of Plane Rigid frames subject to sway, as described in this paper, is basically mechanical. It enables the practical engineer—who has a working knowledge of Hardy Cross' moment distribution and of conventional graphic statics—to solve complicated plane rigid frames involving any number of sways not only for moments, but also (and simultaneously) for axial forces.

The procedure is as close as practicable self checking, i.e. almost every step throughout the operations can be checked and an error detected before it is carried into the next step. A final conclusive check is also available, so that there is no doubt left as to the degree of accuracy of the solution obtained.

The paper deals also with the recognition of the number of independent sway modes a frame possesses.

As the checking procedure forms an important part of the computations, notes pertaining to it are given in Appendix A. In Appendix B the advantage of graphic statics as compared with algebraic methods is shown in mathematical terms. Appendix C proves a formula, given earlier in the paper.

INTRODUCTION

The great advantage of moment distribution is apparent, particularly in cases of highly statically indeterminate structures, because the degree of such indeterminacy is irrelevant. The limitations of moment distribution, however, will be appreciated by the fact, that for frames involving n independent side sways (n + 1) moment distributions have to be performed and

Note: Discussion open until July 1, 1959. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 1943 is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 85, No. ST 2, February, 1959.

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their relative proportions determined for a final superposition.* Further, moment distribution is not capable of checking either the correctness of the introduced fixed end moments, or the consistency of the initial sets of sway moments.

These might contribute to the fact, that there still exists a tendency to design beams and columns with assumed degrees of fixations, rather than to design the rigid frame as a whole. It is true also, that for complex frames involving side sway, there is no easy way to prove conclusively any results obtained, even when deformation due to flexure only is considered.

The only comprehensive flexure check available, is the Mohr moment area procedure, which checks the complete set of final moments independent of the sway or fixed-end moments introduced. However, as the required number of such independent Mohr checks is equal to the degree of statical indeterminacy, it becomes cumbersome and liable to introduce new errors. At its best, it will verify the solution for the moments obtained, but if it fails—and the solution is incorrect—it only confirms this fact, without in any way helping to locate the error (or errors).

If a solution were obtained by slope deflection—or by any other method which treats deflection as the unknown quantity—the complete verification of the results (by any other than the Mohr method) would also be laborious, but the location of an even obvious mistake, will present a still greater task.

The introduction of moment distribution into the design office did not essentially alter this situation, because for frames involving sways, a number of individually conditioned algebraic equations had still to be set up. Although the actual moment distribution computations have self checking properties, the overall results were not capable of being checked by it.

The application of final Mohr checks, as described above, in fact, duplicate some of the work done, if the moment distributions had already been checked individually for rotations at their conclusion.**

Having established a set of final moments, there remains still the task of determining all axial forces, shears and reactions. This had to be done by a further application of "statics". All quantities thus found, would of course be incorrect, if the set of final moments were not the only possible correct one, and a mistake in the application of "statics"—particularly in cases of complex loading—might also easily pass unnoticed, again with the result of an incorrect solution.

The higher the statical indeterminacy, the more imperative it seems therefore, to have a simple and reliable checking procedure incorporating the advantages of Mohr's method, which, at the same time, would extend to all computations including axial forces, shears and reactions. There are, as it seems, no such methods described in the available literature.

As mentioned earlier, it is a characteristic feature of moment distribution that the consistency of impressed sway moments or of fixed-end moments cannot be checked by it. This is due to the fact, that these quantities represent part of the loads introduced. No check by statics will therefore be capable of revealing an error of that type. However, if careful checks are applied*** to introduce these initial moments correctly, an overall check by statics will be conclusive.

---

*Disregarding Prof. Morris' method (see ref. 13, p. 66 and 144).

**See (2) Appendix A.

***See (3) and (4) Appendix A.

Although an elimination of errors is impossible, much valuable time can be saved by the adoption of a process, whereby errors can systematically and speedily be pin pointed.

The method described in the following pages, aims at eliminating sources of errors by minimizing the individually conditioned operations, or the setting up of algebraic equations. Its main advantage perhaps lies in the fact, that most of its steps are capable of being checked when they are first applied, and that the few remaining steps are verified subsequently by a comprehensive overall check—a final Maxwell diagram.

If this diagram "closes", and provided the requirements as stated* were satisfied, all computations including axial and shear forces, reactions and moments must necessarily be correct. If it does not "close", the few unchecked operations must contain the error (or errors), and it will make the task of locating and correcting them much easier. There is now no necessity to apply separate Mohr moment area checks for a verification of the results obtained.

Generally, two different approaches to the sway problem in rigid frames can be traced in the literature. One as described by Matheson** aims at establishing sets of algebraic equations, called sway and deformation equations, without introducing the concept of imaginary supports, the other, as advocated for instance by Kinney,*** sets up individual equations, but evaluates reactions at imaginary supports. Both approaches introduce "cuts"—individually chosen—for establishing equilibrium equations on the various created "free bodies".

These procedures, however, would be different for every frame, and in any case, they would require a thorough understanding of the theory involved. Even in the hands of experts, these procedures tend to introduce mistakes. If one equation only were set up incorrectly, any result obtained would of course be incorrect. Such error would however—at its best—be apparent only after a tedious solution and check had been performed. To locate the error would be a rather difficult task.

The method described in this paper also makes use of cuts, but all "free bodies" here are formed by the joints. For any moment distribution carried out, the sum of moments at any joint equals zero. As neither multiples of sets of end moments nor the summation of such multiples, alters the rotational equilibrium at any joint, two remaining equilibrium conditions only have to be satisfied there. A Maxwell diagram will therefore take care of all statics for every case. As it extends over the whole frame, the Maxwell diagram will be found superior to the methods dealing with only a limited number of individually chosen free bodies. Further, the combination of Maxwell diagram and moment distribution seems ideal, because, while the lack in mathematical accuracy of graphic statics is hardly a disadvantage here, its directness, simplicity and speed are definite advantages.

In the following pages, a short exposition regarding moment distribution of rigid frames subject to side sway is given, and subsequently, the method using moment distribution combined with graphic statics is developed. This procedure is then explained with reference to two worked problems.

---

*See 1 to 4 Appendix A.

**Ref. (4).

***Ref. (8).

It has been stated authoritatively that moment distribution undoubtedly has been the greatest advance in structural analysis during the last decades. If this paper does no more than to give added confidence in the use of moment distribution in complex cases of frames involving sway, it has achieved its intended purpose. It is anticipated, however, that the adoption of the method described, can have economic aspects, both with regards to the saving of highly specialized manpower as well as materials.

Methods of Analysis

The number of independent side sways (n), a frame possesses equals the degree of freedom of the frame when looked upon as pin jointed, and it can be determined by establishing the minimum number of imaginary additional bars necessary to make the "pin jointed frame"* stable (and statically determinate).

Thus, the frames illustrated in the Figs. 1 to 4 have consecutively n = 1, 2, 3 and 4 independent modes of side sway, which in every case is in accordance with the number of imaginary additional bars (shown dotted).

The solution by moment distribution is relatively simple, if n "substructures" can be "created" by applying n imaginary cuts a - a, b - b, etc., each of which cuts parallel or concurrent bars only.

Referring to Fig. 5, it can be seen, that two such parallel bar substructures can be created. The shears at A, D, B and C are calculated from the actual end moments M_{AB} etc. in respective members and, if applicable, from the actual loads acting on members. Thus:

$$\left.\begin{aligned} S_{AB} &= \frac{M_{AB} + M_{BA}}{AB} + S^{\circ}_{AB}; \qquad & S_{DC} &= \frac{M_{DC} + M_{CD}}{DC} \\ S_{BF} &= \frac{M_{BF} + M_{FB}}{BF} + S^{\circ}_{BF}; \qquad & S_{CE} &= \frac{M_{CE} + M_{EC}}{CE} \end{aligned}\right\} \quad (1)$$

where S^{o}_{AB} etc. represent shear forces due to the loads acting on the respective simply supported members of the frame.

Resolving forces in directions perpendicular to the cuts for each substructure, we obtain n equilibrium equations.

$$\Sigma F_a + S_{AB} + S_{DC} = 0; \qquad \Sigma F_b + S_{BF} + S_{CE} = 0 \quad (2)$$

In the above equations ΣF_a, ΣF_a etc. represent components of all external forces in direction a - a and b - b respectively, acting on the free body.

S_{AB} , S_{BF} . . . . etc. are total shears,

M_{AB}, M_{BA} etc. represent actual end moments in respective members of the frame. They are the sum by superposition of moments m_{AB}, m_{BA} . . . . .

---

* Frame to exclude members, which on the rigid frame can be determined by statics.

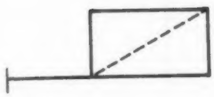


FIG. 1

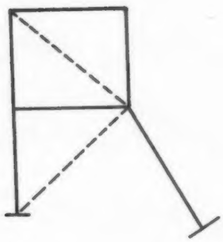


FIG. 2

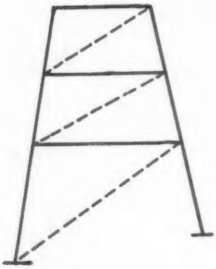


FIG. 3

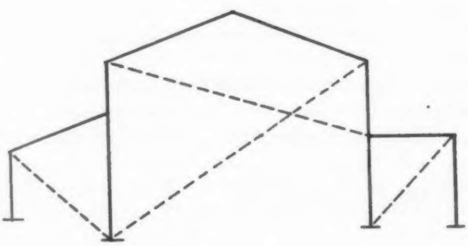


FIG. 4

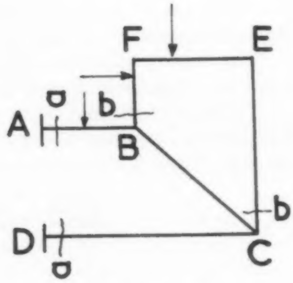


FIG. 5

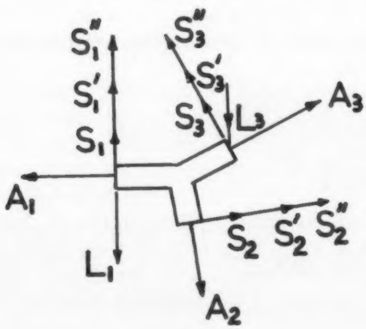


FIG. 6

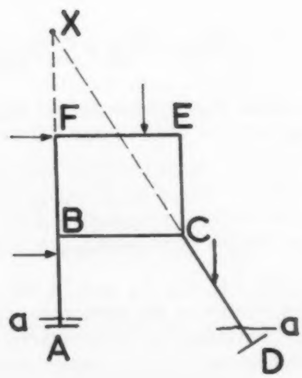


FIG. 2a

etc. obtained as a result of moment distribution due to loads only (no sway), and of a fraction or multiple (K) of moments m'_{AB}, m'_{BA}, m''_{AB}, m''_{BA} . . . . . etc., obtained as a result of moment distributions due to the various independent side sways. Thus:

$$\left.\begin{aligned} M_{AB} &= m_{AB} + K_1 m'_{AB} + K_2 m''_{AB} + \ldots \\ M_{BA} &= m_{BA} + K_1 m'_{BA} + K_2 m''_{BA} + \ldots \\ M_{DC} &= m_{DC} + K_1 m'_{DC} + K_2 m''_{DC} + \ldots \\ &\text{etc.} \end{aligned}\right\} \qquad (3)$$

Generally, each line of Eqs. (3) will have (n + 1) terms. If the expressions (3) are combined with (1) and (2), as many linear equations in K_1, K_2 . . . . etc. will result, as the frame possesses independent side sways. From these the coefficients K_1, K_2 etc. are readily obtained.

The actual Moments M_{AB} etc., as well as the actual shears S_{AB} etc. can now be calculated from (3) and (1).

If the bars in the above referred imaginary cuts were not parallel, the equilibrium equations corresponding to (2) would contain components of axial forces. Thus for cut a - a along the base of frame Fig. 2 (see also Fig. 2a)

$$\left.\begin{aligned} \Sigma H + S_{AB} + S_{DC_a} + A_{DC_a} &= 0 \\ &\text{etc.} \end{aligned}\right\} \qquad (2a)$$

where the subscript a expresses the fact that the force component in direction a - a is considered. It can be shown (see Appendix C) that:

$$A_{CD} = a_{CD} + c_0 m_{CD} + K_1 c_1 m'_{CD} + K_2 c_2 m''_{CD} + \ldots \text{etc.} \qquad (4)$$

where the four terms on the right hand side of the equation represent consecutively axial forces in bar CD caused by

loads on the pin jointed frame only
end moments in bars meeting at C due to loads
end moments due to sway 1, and
end moments due to sway 2

In Eq. (4), C_0, C_1 and C_2 etc. are constants depending on the loading and the geometry of the structure, and m_{cd}, m'_{cd}, m''_{cd} etc., are moments as before. Provided these constants could be evaluated, Eqs. (2a) would again represent the n linear equations in K_1 to K_n. This approach however is no more direct.

On the other hand, by taking moments about a common intersection point X of the cut members, an equilibrium equations, which will not contain axial forces, will be obtained for each of the n sub-structures.

Referring again to Fig. 2a, for cut a - a and with X as intersection point

$$\Sigma M_X = 0 = \left[\frac{M_{DC}+M_{CD}}{DC} + \overset{\circ}{S}_{DC}\right] DX + M_{DC} + \\ + \left[\frac{M_{AB}+M_{BA}}{AB} + \overset{\circ}{S}_{AB}\right] + M_{AB} + \Sigma\, Fd \tag{2b}$$

where ΣFd represents the sum of all external moments about X, and AB etc. are the lengths AB etc. Otherwise, the nomenclature is the same as before.

If the moments M are expressed by Eq. (3), the above Eq. (2b) will become a linear expression in K_1, K_2, and generally for n such cuts, a number of equations equal to the n independent sways will result.

Frames as referred to above could therefore be solved without first evaluating any axial forces, and without the concept of imaginary supports and reactions. For a more general case of a rigid frame however, the approach becomes impracticable, without evaluating at least some of the axial forces in the process of the analysis (see ref. 8 page 422).

In the method by Matheson (see ref. 4) scores of individually conditioned algebraic sway and deformation equations have to be established.

These would have to be prepared very carefully indeed, as a mistake in a sway equation, for instance, could not be discovered by substituting the final moments obtained back into the sway equations. Thus, it might easily happen, that a correct algebraic solution was found for the wrong sway equations, giving the designer an erroneous and dangerous conclusion.

Fortunately, there is no need to evaluate algebraically any of such complex expressions, or to set up sway equations, if the problem is approached in a different manner. This latter approach will now be discussed. It will be seen that by its use, any plane rigid frame can be solved almost mechanically.

The Semi-Graphical Method

By introducing on the frame, in suitable positions, as many imaginary roller supports* as there are independent sway modes, the frame will be prevented from swaying. In the case of one initial arbitrary sway displacement, by the use of these imaginary roller supports, no further swaying will be possible.

The frame, which is now loaded by either the given loads or by an introduced displacement in accordance with the assumed side sway under consideration, will—after relaxation (rotation) of all joints—take up an equilibrium condition. Each joint must, then, be in equilibrium. The actions operating on it will be (see Fig. 6):

(1) Moments adding up to zero,
(2) Forces L_1, L_2 etc. due to loads L on those bars meeting at the joint. (L_2 is here assumed zero)
(3) Shears due to the end moments on all bars meeting at the joint
(4) Axial forces and (where applicable) reactions.

*Direction of roller bed is arbitrary, excluding some critical cases.

The forces under (2) are easily evaluated, as they represent the negative reactions, if the joints were considered "pin jointed" (L_1 and L_3 in Fig. 6). The shears under (3) can be subdivided into those produced by

(a) Moments resulting from moment distribution due to loads only (no sway),
(b) Moments resulting from moment distribution due to a particular sway displacement,
(c) Moments resulting from moment distribution due to another independent sway displacement,

etc.

Shears (3a, 3b), etc. are obtained by dividing the algebraic sum of the distributed end moments by the length of the bar in question. Thus there will be at each joint from (2) and from (3a), (3b), (3c), etc. in each case, a resulting force R with known components R_x and R_y.

Now, for each joint there could be written two equations of equilibrium. If j is the number of joints—not counting fixation joints—and n is the number of independent sidesways, in accordance with the rule laid down earlier, there will be 2j - n bars in the frame (also 2j - n unknown axial forces), and n imaginary reactions at the assumed points preventing sway.

The number of unknown forces 2j - n + n equals therefore the number of equations which can be written. Consequently, the frame with its imaginary supports can be treated like a statically determinate pin jointed truss, and for each set of forces according to (2), (3a), (3b), etc., a Maxwell diagram* can be drawn giving at once all axial forces and reactions (at the imaginary roller supports) for each of the loading conditions referred to above.

As the frame has in fact no roller supports preventing sways, the magnitude of each independent (and arbitrary chosen) side sway has to be modified in such a manner, that upon adding up of the several reactions at each imaginary fixation point—due to the various loadings—such sums will equal zero. From this condition the coefficients K_1, K_2 etc. are determined, as there are as many equations as coefficients K.

Thus:

Reaction at point 1
Reaction at point 2

$$\left.\begin{aligned} R_1 = 0 &= r_{10} + K_1\, r_{11} + K_2\, r_{12} + \cdots \\ R_2 = 0 &= r_{20} + K_1\, r_{21} + K_2\, r_{22} + \cdots \end{aligned}\right\} \quad (5)$$

where

r_{10} is the reaction at roller support 1 due to

(I) loads acting on the "pin jointed frame" plus
(II) forces due to moment distribution for "no side sway" and, for instance

r_{12} is the reaction at roller support 1 due to sway 2

etc.

From Eqs. (5) all K values can be calculated. Then, with the notation as in Eq. (3), the moment in any bar, say bar CD, will be

*In some cases special procedures for inst. virtual displacements have tc be used, before a force plan can be drawn.

$$M_{CD} = m_{CD} + K_1 m'_{CD} + K_2 m''_{CD} \tag{3a}$$

and the axial force in bar CD will be

$$A_{CD} = a^{o}_{CD} + K_1 a'_{CD} + K_2 a''_{CD} \tag{4a}$$

where

a^{o}_{cd} is the axial force in bar CD due to loads on the pin jointed frame, and to moment distribution "no sway",

a'_{cd} is the axial force in bar CD due to end moments from sway 1,

a''_{cd} is the axial force in bar CD due to end moments from sway 2.

etc.

Now the values for a^{o}, a', a'' etc. are readily available from the various Maxwell plans, and with the K values known, the moments and the axial forces can be calculated using Eqs. (3a) and (4a).* The shears can be calculated as in Eq. (1).

These results, however, would require to be verified before they could be relied upon. The set of moments M_{CD} etc., could be checked by the application of an appropriate number of independent Mohr checks. However, if one of these only should happen not to be satisfied, neither the moments nor shears and axial forces, as found earlier, could be relied upon, and the tracing of an error could be difficult.

The Final Maxwell Diagram as Check

As an alternative to the procedure just outlined, we can with advantage use Eq. (3a) only, and so establish a complete set of final moments. If from these, all shears are determined, and the forces acting on the joints (from the given loading) are added where applicable, a final set of joint forces will thus be obtained.

If the set of final moments is to be the only correct one applying to the frame and its loading, the final set of joint forces obtained must also be correct. Consequently, a consistent final Maxwell plan must result with all reactions at imaginary supports equal to zero.

The final Maxwell plan, therefore not only produces all axial forces (as an alternative to using Eqs. (4a)), but it provides also a decisive final check for the degree of accuracy of the whole computation. This is true in particular, as geometric checks—taking into account the physical properties of the bars—were already applied with the individual moment distribution computations earlier (see 3 Appendix A).

The method described, can equally be applied to frames defined with reference to Fig. 5. Although the analytical methods might be more direct in this case, they would not provide the same sequence of checks, which forms a characteristic feature of the semi-graphical analysis described here.

As a number of Maxwell plans are required for the same frame, but with varying forces acting at the joints, the use of Bow's notation may produce

*Comparing Eq. (4a) term by term with Eq. (4), the coefficients of Eq. (4) (c_o, c_1, c_2 etc.) could now easily be determined.

different symbols for forces in the same member. Unless special care is taken, it might be advisable to dispense with Bow's notation in favour of a system whereby the members are denoted AB, BC, CD, etc., with A, B, C etc. being joints.

The method will be illustrated with reference to (A) a simple portal frame loaded with a horizontal concentrated force, and (B) to a complex cantelever type frame,* ninefold statically indeterminate, loaded by a series of concentrated forces and distributed loading as shown (see Figs. 7a and 7b).

Sign Convention

In the following problems, the sign convention as under will be used: Anticlockwise Moments on ends of bars are positive, and shears are positive if an imaginary roller between the cut faces of a bar would rotate clockwise.

A. Portal Frame with Side Load

The frame loaded as illustrated in Fig. A1 is not subject to fixed end moments, the deformations being due to sidesway only. The moment distribution for sidesway is carried out in Fig. A2, where due to symmetry, instead of writing "+1" for sway moments at both columns tops, the left hand fixed-end moment only is considered and the final results are obtained by superposition with appropriate values from the right hand side of the table. An exact solution is obtained (see Fig. A3). This computation is checked as follows:

$$\Theta_{BA} \sim \frac{-40 - \frac{-20}{2}}{1} = -30 \quad , \quad \Theta_{BC} \sim \frac{-60 - \frac{-60}{20}}{1} = -30$$

and

$$\Theta_{AB} = \Theta_{DC} = 0,$$

as the rotations at a joint are proportional to the total change in moment at the near side less one half of the total change in moment at the far side of a prismatic member divided by its stiffness.

The shears are calculated and applied on the joints as external forces. Subsequently, a Maxwell diagram is drawn, assuming an imaginary roller support—taking horizontal thrust—at point C (see Figs. A3 and A4). The reaction at the imaginary support is found to be R = OP = 28. The coefficient K is found from the condition that 5 - 28K = 0 as $K = +\frac{5}{28}$; using Eqs. (3a), the real moments are therefore:

$$M_{AB} = M_{DC} = \frac{+5}{28} \times \cdot 80 = +14\frac{2}{7}\,; \qquad M_{BA} = +10\frac{5}{7}$$

*Dimensions and loads were applied from the practice of a consulting engineer.

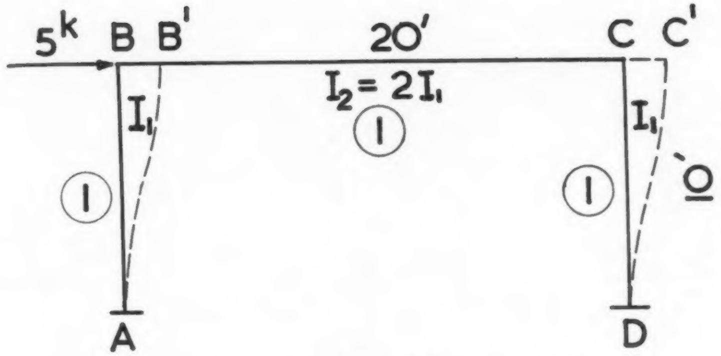

5k
B
B'
20'
C
C'
I2 = 2I1
1
I1
1
1
O
A
D

FIG. A1.

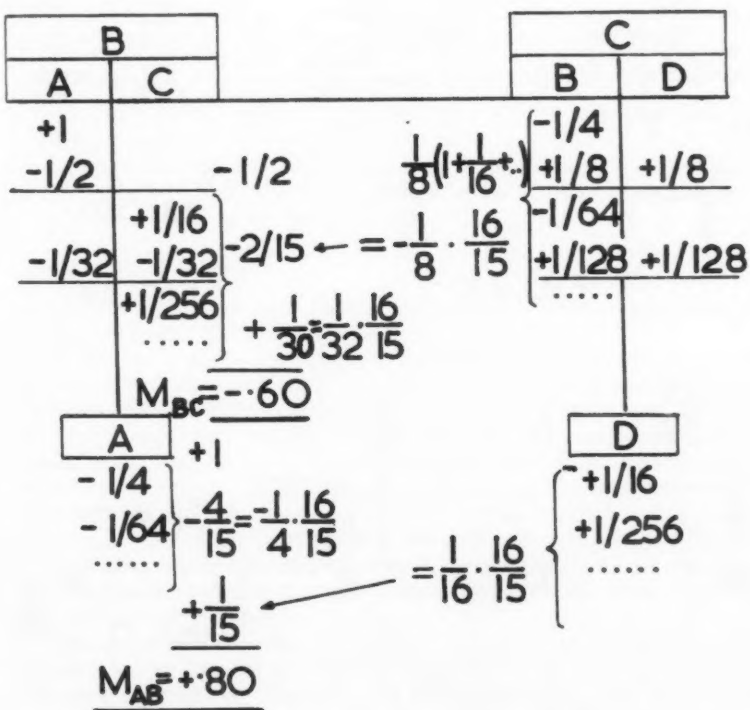

B
A
C
C
B
D
+1
-1/2
-1/2
-1/4
1/8(1+1/16+..)
+1/8
+1/8
+1/16
-1/64
-1/32
-1/32
-2/15 = -1/8 · 16/15
+1/128
+1/128
+1/256
+ 1/30 = 1/32 · 16/15
......
MBC = -·60
A
D
+1
-1/4
+1/16
-1/64
-4/15 = -1/4 · 16/15
+1/256
......
= 1/16 · 16/15
+1/15
MAB = +·80

FIG. A2

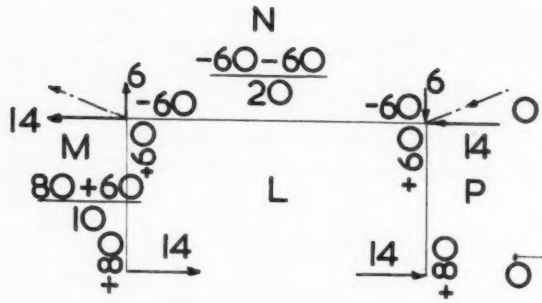


FIG. A3

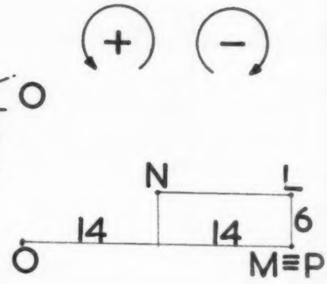


FIG. A4

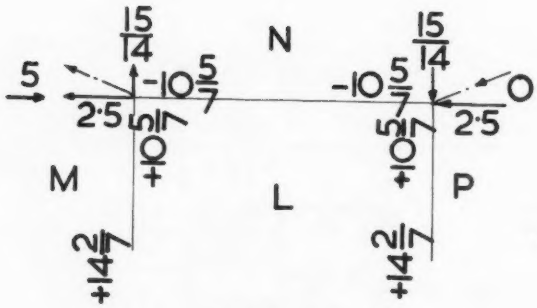


FIG. A5

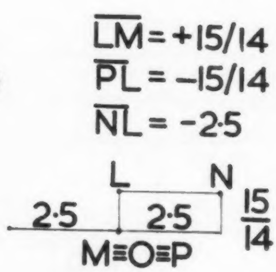


FIG. A6

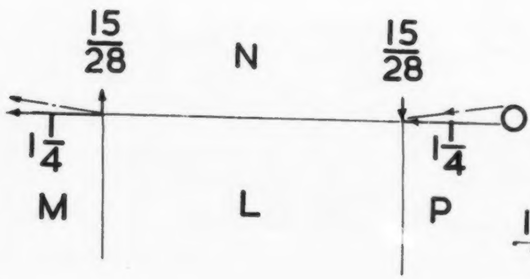


FIG. A6a

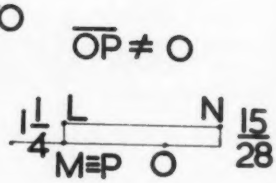


FIG. A6b

$$M_{BC} = M_{CB} = \frac{+5}{28} \times -60 = -10\tfrac{5}{7}; \qquad M_{CD} = +10\tfrac{5}{7}$$

and the shears

$$S_{AB} = S_{DC} = \frac{14\frac{2}{7} + 10\frac{5}{7}}{10} = 2{\cdot}5;$$

and

$$S_{BC} = S_{CB} = \frac{-10\frac{5}{7} - 10\frac{5}{7}}{20} = \frac{-15}{14}$$

are acting on the joints as shown in Fig. A5. The final Maxwell diagram (Fig. A6) gives all axial forces and shows the reaction at the imaginary support OP to be zero. The solution is therefore correct. Had an error occurred for example in one of the above shear computations, so that the equation for K would have been incorrect, or if for instance one of the equations (3a) were set up incorrectly, the final Maxwell diagram would not "close". In Figs. A6 a and b, it is assumed that an erroneous value say $K = \frac{5}{56}$ (instead of $K = \frac{5}{28}$) were used in the determination of the final set of moments. The reaction at C or OP in Fig. A6 b equals now $2.5 \neq$ zero, and consequently the results cannot be correct.

Although the above problem can obviously be solved quicker by other means, its inclusion here serves to prove, that the method is applicable to simple frames as well as to complex ones.

B. Complex Rigid Frame

The plane rigid frame with dimensions and loads as shown in Fig. B1 is to be analysed by the method described.

The loads on the joints, as produced by the given loading on the pin jointed frame were first calculated and the results entered in Fig. B2. Fig. B3 shows the stiffness and distribution factors applying to the various members of the frame. The tables (Fig. B4) show values for the simply supported moment diagrams, while Fig. B5 shows the computation of the fixed End Moments for the three loaded spans. These must be checked (see 3, Appendix A).

Using these fixed-end moments, the moment distribution due to loading only (no sway) was carried out yielding the end-moments as shown in Fig. B6. Compatible sway moment sets were then determined in the conventional manner for the two sway modes with imaginary horizontal restraints at points D and E. (Figs. B7 a and B9).

The results of sway 1 were checked by the method as explained in point 4 of Appendix A (see Fig. B7 b). Subsequently the two sway moment distributions were carried out. Figs. B8 and B10 show the sets of moments before and after balancing only. The joint forces were then determined. All moment distributions were checked for equal rotation at joints (see Table Fig. B11).*

*Strictly, as there are 8 bars, 16 angular rotations would have to be checked for each moment distribution, corresponding to 16-5=11 required independent continuity conditions for the 11-fold stat. indet. joint positioned rigid frame.

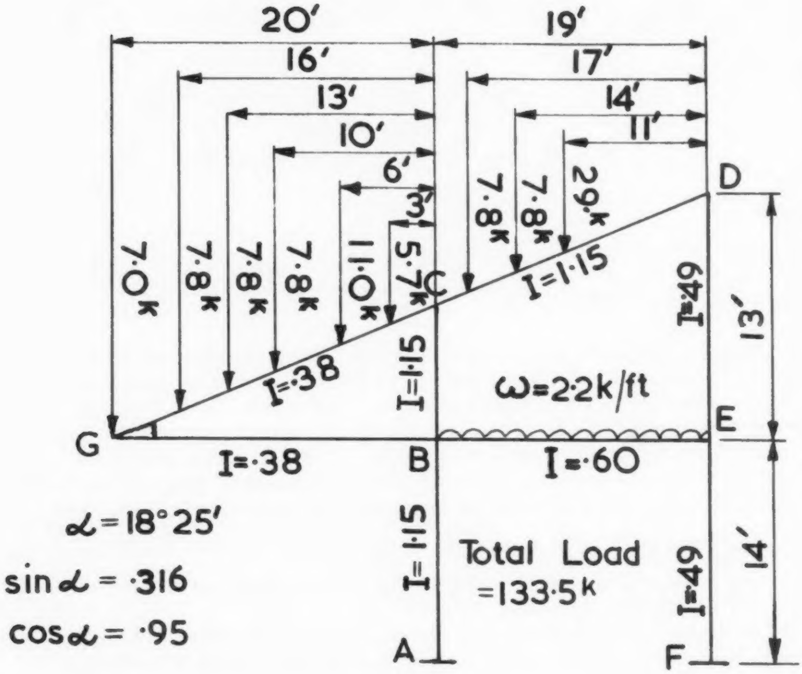
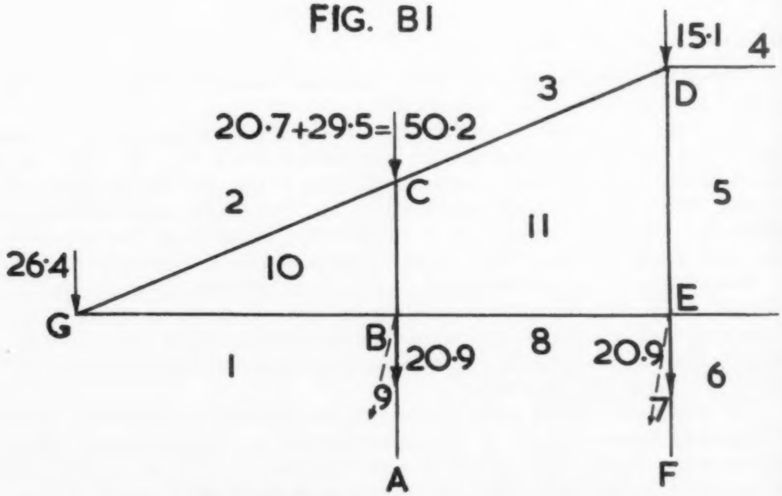
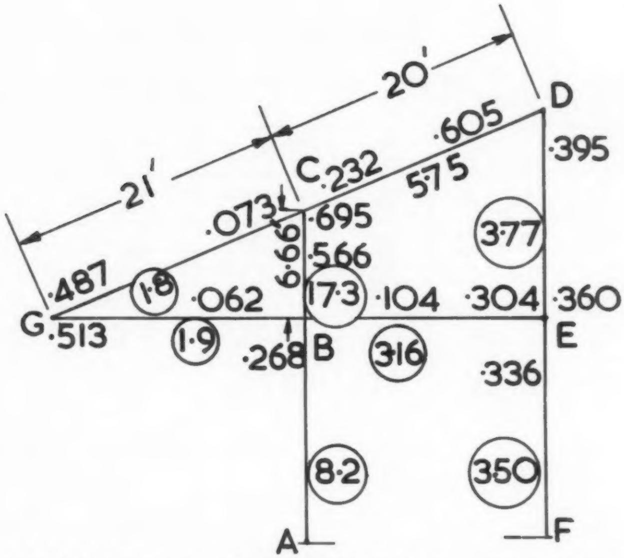


FIG. B1



Check: $26.4 + 50.2 + 15.1 + 19 \times 2.2 = 133.5$ ✓

FIG. B2



STIFFNESS & DISTRIBUTING FACTORS

FIG. B3

GC.

LOCATION	20'	16'	13'	10'	6'	3'	0
M_o	0	+77·6	+112·6	123·8	+107	+62·1	0

CD.

LOCATION	19'	17'	14'	11'	0'
M_o	0	+59·0	+124	+166	0

BE.

LOCATION	19'	14·25'	9·5'	4·75'	0
M_o	0	+74·5	+ 99	+74·5	0

SIMPLY SUPPORTED MOMENTS FOR SPANS (GC, CD, & BE)

FIG. B4

Span G-C = 20′ hor.

Load P, kips	a ft	b ft	$\frac{P}{l^2}$	$\frac{Pa^2b}{l^2}$	$\frac{Pb^2a}{l^2}$
5·7	17	3	·0142	12·3	2·17
11·0	14	6	·0275	32·4	13·85
7·8	10	10	·0195	19·5	19·5
7·8	7	13	·0195	12·4	23·0
7·8	4	16	·0195	5·0	20·0
7·0	0	20	·0175	0	0
			Σ =	− 81·6	+78·5
				in kip ft.	
				FM_{CG}	FM_{GC}

Span C-D = 19′ hor.

Load	a	b	$\frac{P}{l^2}$	$\frac{Pa^2b}{l^2}$	$\frac{Pb^2a}{l^2}$
7·8	2	17	·0216	1·38	12·5
7·8	5	14	·0216	7·56	21·2
29·0	8	11	·080	56·3	77·3
			Σ =	+65·2	− 111·0
				in kip ft.	
				FM_{CD}	FM_{DC}

Span B-E

$$FEM = \frac{Pl}{12} = \frac{2{\cdot}2 \times 19 \times 19}{12} = 66{\cdot}1$$

FIXED END MOMENTS

FIG. B5

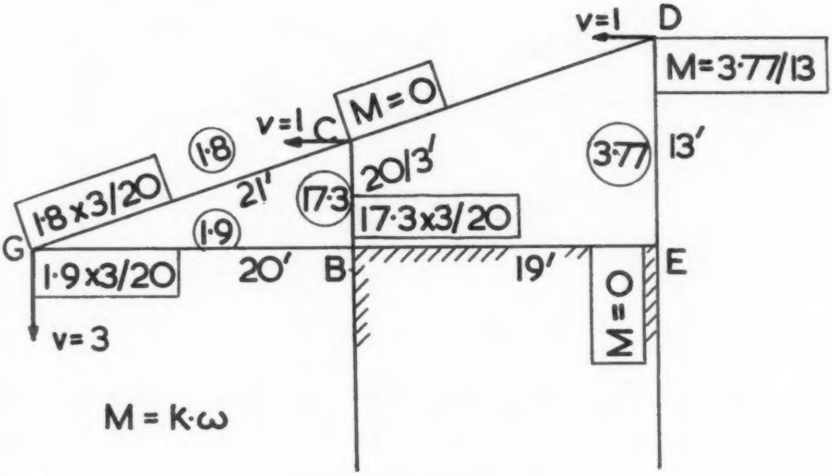


FIG. B7b

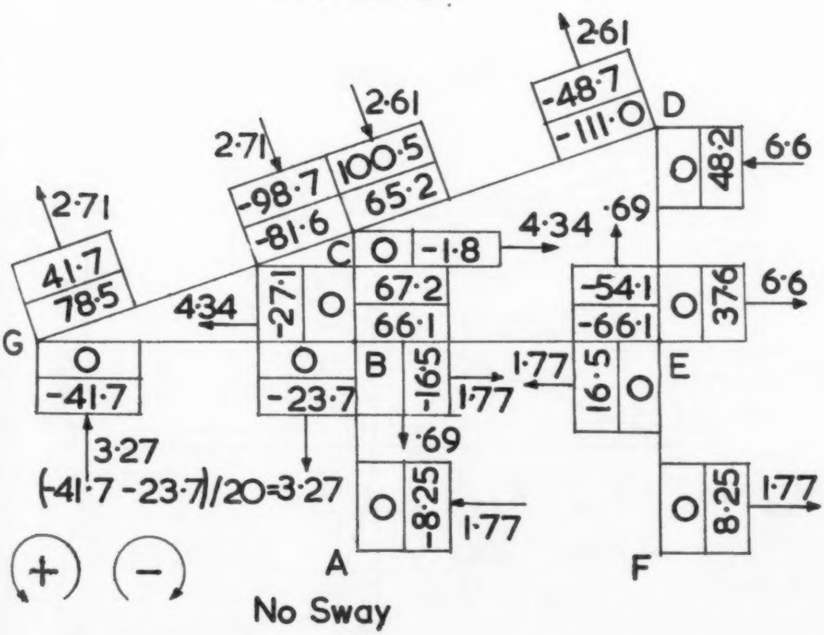
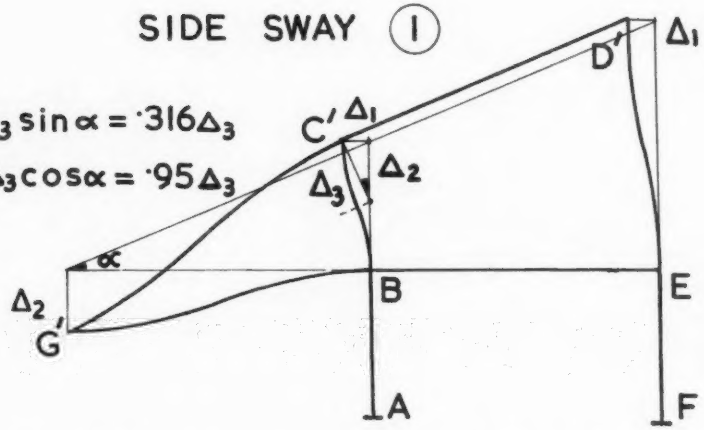


FIG. B6

SIDE SWAY (1)

$\Delta_1 = \Delta_3 \sin\alpha = {\cdot}316\Delta_3$

$\Delta_2 = \Delta_3 \cos\alpha = {\cdot}95\Delta_3$

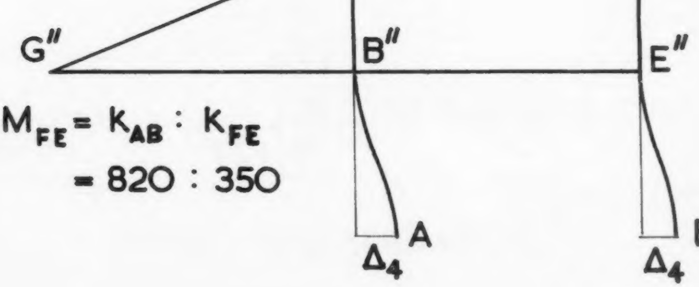


$$M_{DE} : M_{CB} : M_{CG} : M_{BG} = \Delta_1 \frac{k_{DE}}{l_{DE}} : \Delta_1 \frac{k_{CB}}{l_{CB}} : \Delta_3 \frac{k_{CG}}{l_{CG}} : \Delta_2 \frac{k_{GB}}{l_{GB}}$$

$$= 91{\cdot}5 : 822 : 85{\cdot}7 : 90{\cdot}2$$

FIG. B7a

SIDE SWAY (2)



$$M_{AB} : M_{FE} = k_{AB} : k_{FE}$$

$$= 820 : 350$$

FIG. B9

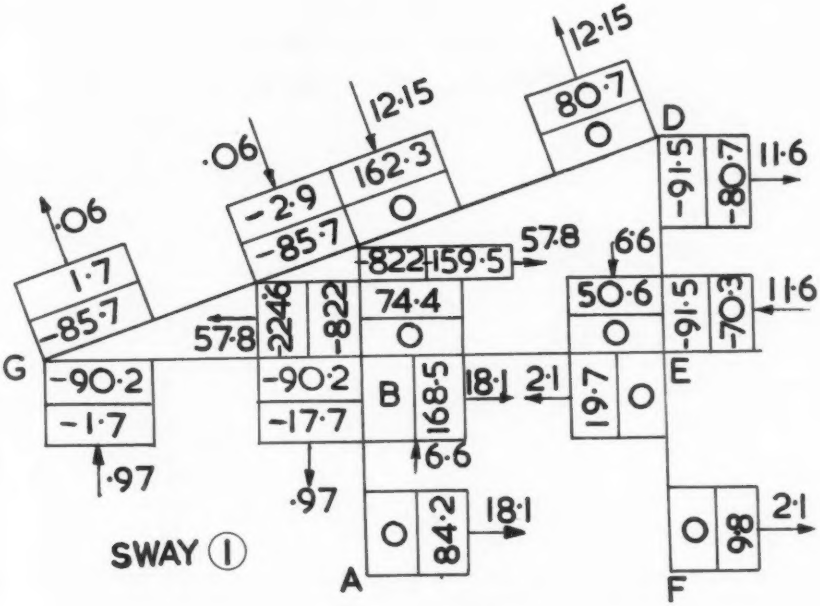


FIG. B8

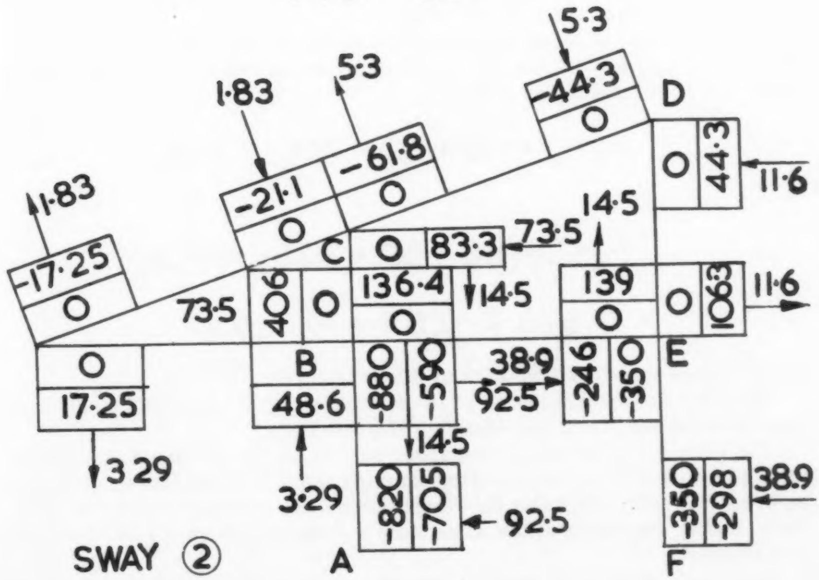


FIG. B 10

ROTATIONS

$$\theta_{AB} = (M_{AB} - \tfrac{1}{2}\Delta M_{BA})/K$$

JOINT		NO SWAY	SWAY ①	SWAY ②
JOINT-B	θBG	−1.50	14.9	21.0
	θBC	−1.51	15.3	21.0
		−1.54	15.5	21.2
	θBA	−1.51	15.4	21.0
JOINT-C	θCG	.72	21.7	−6.95
	θCB	.68	20.0	−6.92
	θCD	.68	20.9	−6.90
JOINT-E	θEB	3.63	4.24	22.4
	θED	3.6	4.19	22.4
	θEF	3.54	4.23	22.3

FIG: B11

The joint force diagrams arrived at by using sets of end moments, according to the three moment distributions, are shown in Figs. B12 to B14, while Fig. B15 to B18 show the Maxwell diagrams applying to the statically determinate pin-jointed frame with assumed imaginary roller supports for horizontal thrusts at points D and E of the frame.

The reactions at these imaginary supports, as found graphically by the Maxwell diagrams, provide the coefficients in the two simultaneous linear equations in K_1 and K_2 as follows:

$$H_D = 0 = -16 + 80 - 74K_1 + 88K_2 \tag{1}$$

$$H_E = 0 = +16 - 80 + 94.6K_1 - 220K_2 \tag{2}$$

and

$$K_1 = +1.06; \qquad K_2 = +.164$$

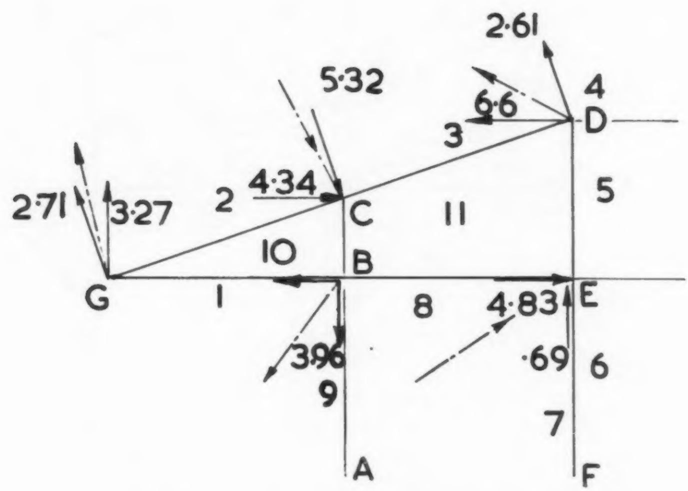
which values satisfied Eqs. (1) and (2).

Note that the determination of the coefficients "16" and "80" could have been carried out, using one Maxwell diagram only.

The problem could now be regarded solved, as the moments, shears and axial forces would readily be obtained using Eqs. (3a) and (4a) as described before. However, the following computations are devised to give the same results, and to represent at the same time a complete check for all these quantities obtained.

The establishment of the final sets of moments and joint forces is shown in Fig. B19. The final Maxwell diagram Fig. B20 takes into consideration the

FORCES ON JOINTS
NO SWAY
As produced by the distrib. End Moments on Bars



CHECK $\Sigma H = 0 = 1{\cdot}77 - 1{\cdot}77$

$6{\cdot}6 \underleftarrow{- 4{\cdot}34} + 4{\cdot}\underrightarrow{83} - 2{\cdot}57 = 0$ ✓

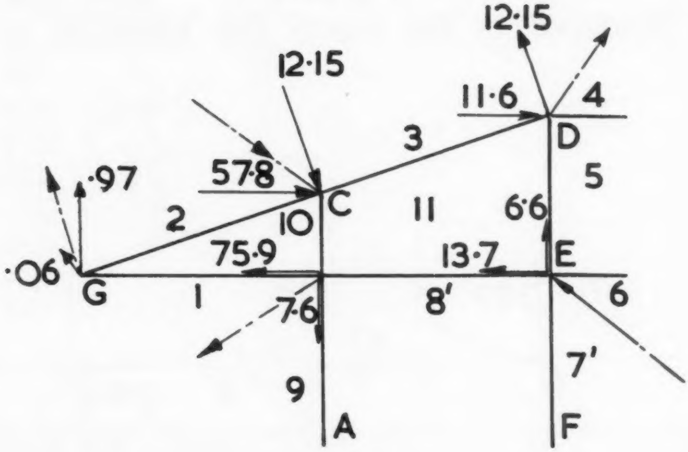
from MAXWELL PLAN

$16 - 16 = 0$ ✓

FIG. B12

FORCES ON JOINTS
SWAY (1)

As produced by the distrib. End Moments on Bars



Check $\Sigma H = 0 = 18{\cdot}1 + 2{\cdot}1 = 20{\cdot}2$

$75{\cdot}9 + 13{\cdot}7 - 57{\cdot}8 - 11{\cdot}6 \quad 20{\cdot}2$ ✓

From Maxwell Plan:

$94{\cdot}6 - 74 = 20{\cdot}6$ ✓

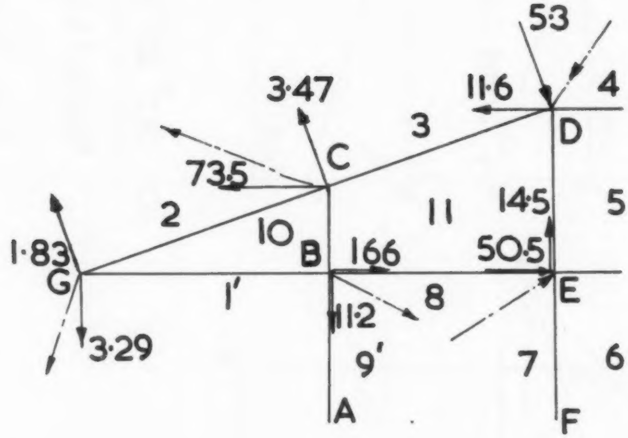
FIG. B13

forces from Fig. B19 and those due to loads (Fig. B2). It shows all axial forces to scale, and the imaginary reactions are found to be negligible (distances 4 - 5 and 5 - 6 in Fig. B20), indicating that the solution can be regarded as correct.

The final bending moment diagram is shown in Fig. B21, where the ordinates appear on the tension side of the members. Fig. B22 shows the axial force diagram. In this diagram, corrections for the axial forces in the inclined members had to be made, as equivalent concentrated forces had previously been assumed at the joints.

In Fig. B23, it is assumed that one concentrated force P only acts on an inclined bar AB. The difference in magnitude of the axial forces at cuts 1 and

FORCES ON JOINTS
SWAY ②
As produced by the distrib. End Moments on Bars



Check $\Sigma H = \overrightarrow{92{\cdot}5 + 38{\cdot}9} = 131{\cdot}4$

$-\underrightarrow{166} - 50{\cdot}5 + \underleftarrow{73{\cdot}5 + 11{\cdot}6} = \underline{-131{\cdot}4}$ ✓

From Maxwell Plan:

$220 - 88 = 132{\cdot}0$ ✓

FIG. B14

2 must equal P sin α. The axial force in bar AB, as found from a Maxwell diagram, would therefore require to be increased between A and force P by a correction ΔA, while between B and force P, it would have to be decreased by an appropriate amount. The corrections are obtained from the two conditions set out below

$$\Delta_A + \Delta_B = P \sin\alpha$$

and

$$\frac{\Delta_A}{\Delta_B} = \frac{R_A}{R_B} = \frac{b}{a}$$

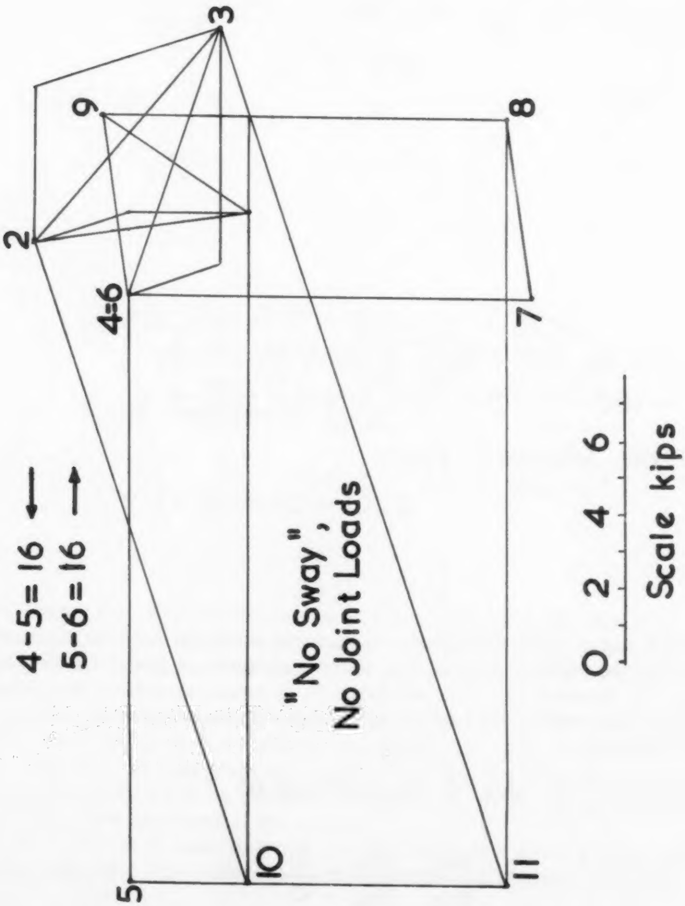


FIG. B15

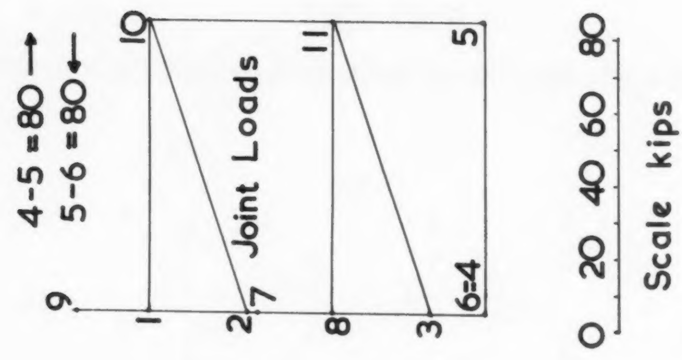


FIG. B16

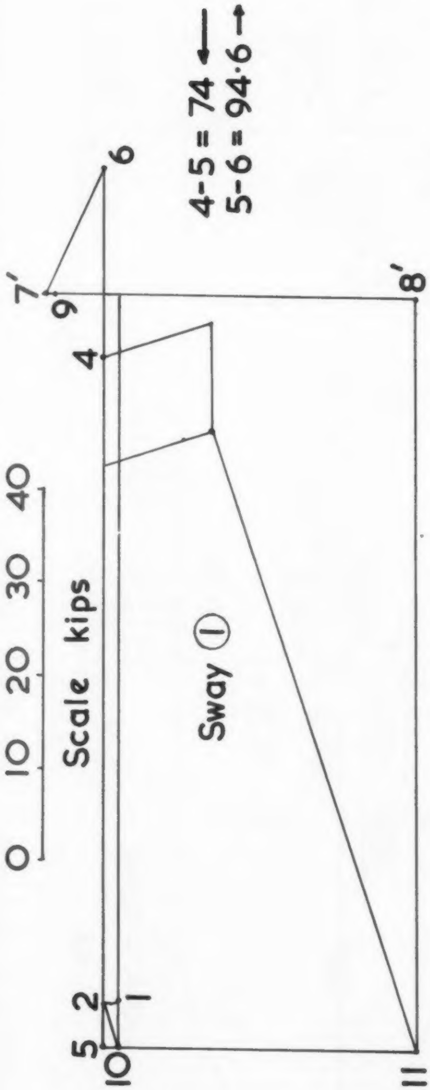


FIG. B17

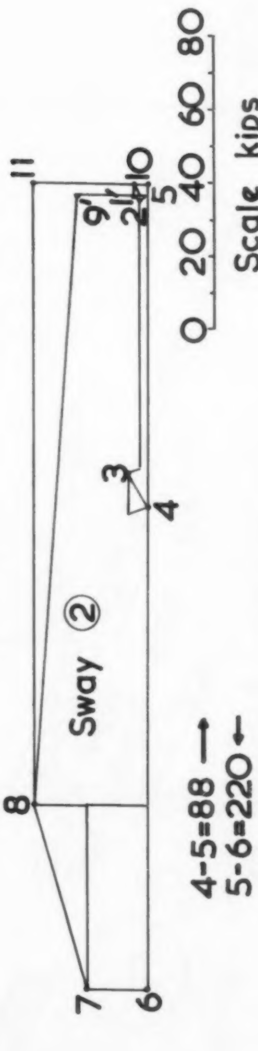


FIG. B18

Final Moments on Members & Forces on Joints due to Bending (ft kips, kips)

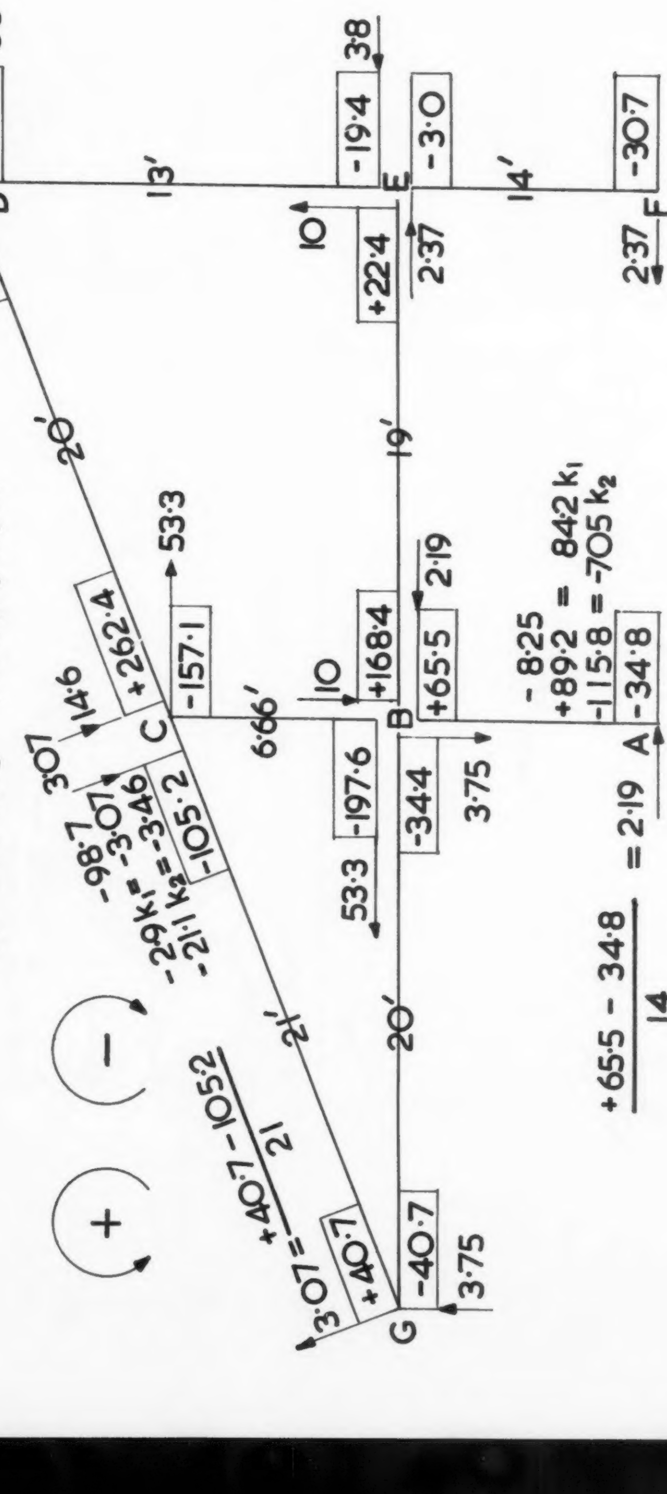


FIG. B19

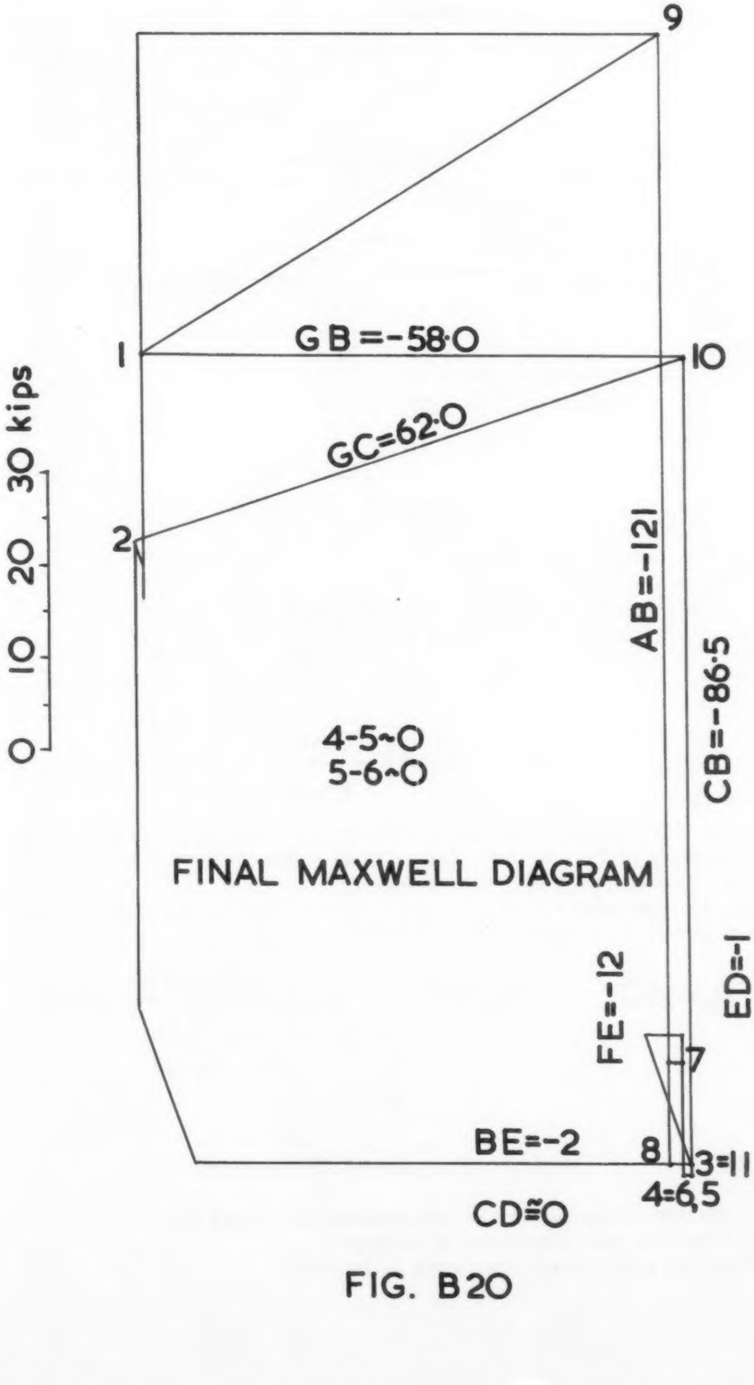
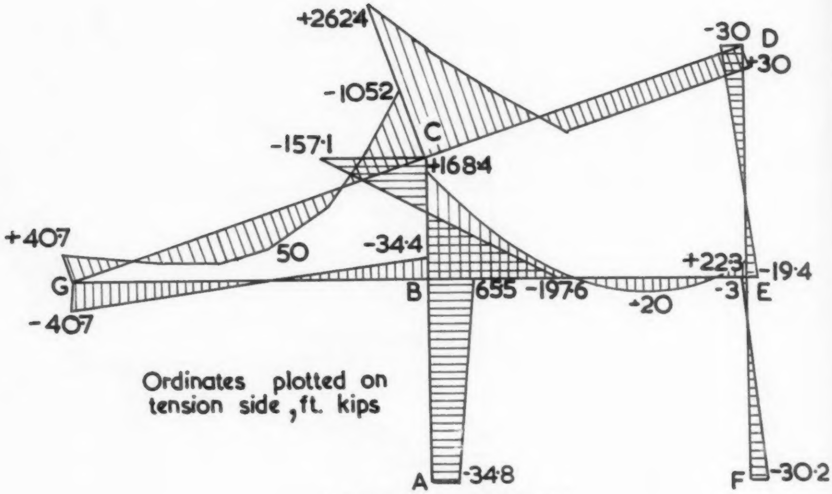

9
1
GB=-58·0
10
GC=62·0
2
30 kips
20
10
0
AB=-121
CB=-86·5
4-5~0
5-6~0
FINAL MAXWELL DIAGRAM
FE=-12
ED=-1
7
BE=-2
8
3=11
4=6,5
CD≅0

FIG. B20



MOMENT DIAGRAM
FIG: B21

so that

$$\Delta_A = \frac{P \sin\alpha}{1 + a/b}$$

and

$$\Delta_B = \frac{P \sin\alpha}{1 + b/a}$$

If there are more forces acting instead of P only, their resultant P_R will take the place of P in the formulae, with a_r and b_r being the distances to its line of action from A and B respectively.

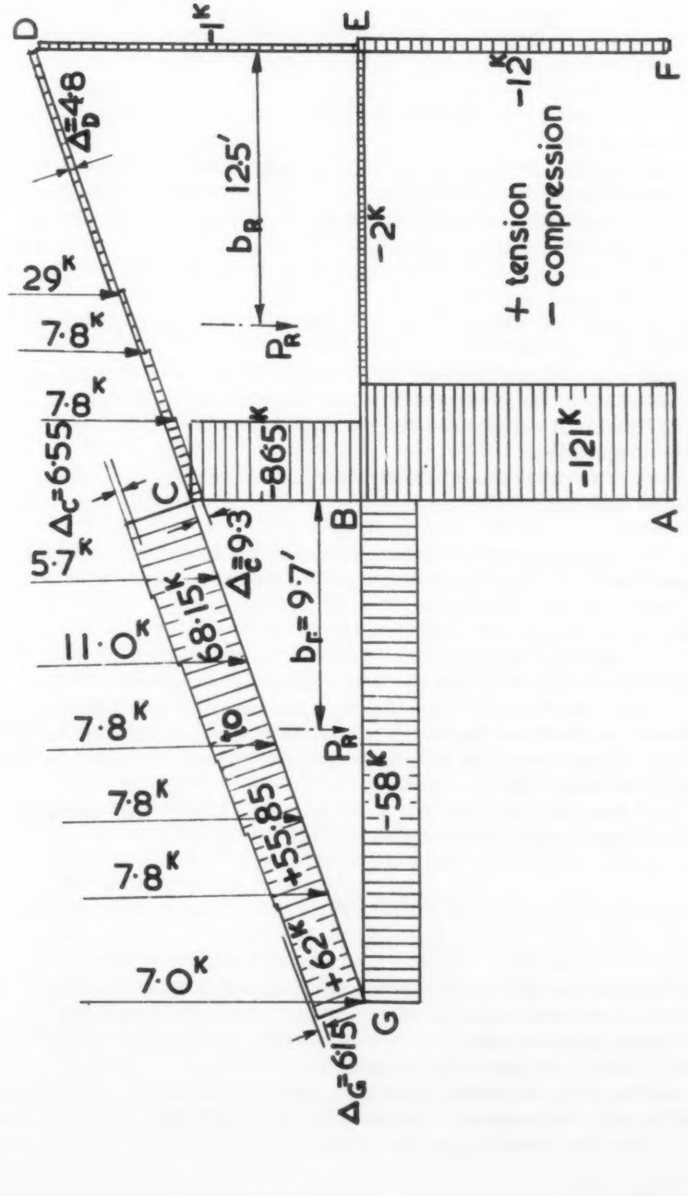
The average axial forces, as obtained from the Maxwell plan Fig. B20, have therefore to be corrected as follows: for bar GC:

$$P_R = 40.1 \qquad b_R = 9.7 \qquad \Delta_C = \frac{40.1 \times .316}{1 + \frac{9.7}{10.3}} = 6.55$$

etc.

Review of Recommended Procedure

(1) Determine the number of independent side sways (n)
(2) Establish joint forces due to loading
(3) Locate n imaginary restraints as follows:



AXIAL FORCE DIAGRAM KIPS

FIG. B22

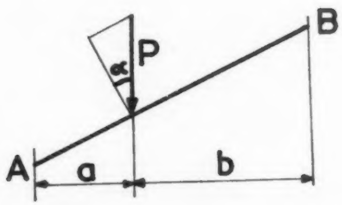


FIG. B 23

Regarding the rigid frame as pin jointed, and starting at a joint where only 2 unknown bar stresses exist, investigate a suitable sequence of joints by which a Maxwell diagram could be arrived at.* It will be found necessary to introduce a total of n imaginary reactions at some of the joints in order to make possible a Maxwell diagram. This determines the imaginary supports

(4) Establish the fixed-end moments from loads where applicable and check them (see (3) Appendix A)

(5) Imagining one restraint temporarily displaced, (without rotation of joint) establish a set of sway moments. Repeat for other restraints. This will yield all compatible sets of sway moments. Check according to note 4 Appendix A.

(6) Apply (n + 1) moment distributions with initial moments according to (4) and (5) and check all end moments by rotation checks. (see 2 Appendix A)

(7) Divide the algebraic sum of end moments by the length for each bar, and apply force couples at joints perpendicular to the bar (sense of rotation same as that of algebraic sum of end moments.)

(8) Combine joint forces due to (2) and (4)

(9) Draw n + 1 Maxwell diagrams and find restraints at all supports. These will yield the coefficients for the n simultaneous equations.

(10) Solve n linear equations for K_1, K_2 etc. and check by substitution.

(11) Determine the final end moments for all bars by superposition according to Eq. (3) and establish a final set of joint forces (inclusive of joint forces found under (2)).

(12) Draw final Maxwell diagram and scale all axial forces and reactions. (The reactions at imaginary supports should be zero).

APPENDIX A

Checks

In order to reduce the time required for any analytical investigation, it is justified to first make an analysis of all possible checks and note their inherent limitations. A systematic process of checking should then be established and followed throughout the computations.

The observation of the following notes on checks is therefore recommended, as without an effective system, it is very difficult to locate an error, even if it is obvious, that the result cannot be correct.

---

*See footnote Page 114.

(1) There will always be certain data for which no checking system applies (given dimensions, loads, cross sections, stiffness, etc.). Particular care must be taken to introduce these correctly. Stiffness and distribution factors should be written into the moment distribution tables in one operation (using for instance carbon paper). This will avoid errors in using wrong coefficients in the various moment distributions.

(2) The moment distribution provides a rotation check, i.e. where members meet, the angular rotations of all members must be the same. Similarly, as changes in lengths are not taken into consideration, certain deflections will have to be equal for geometric reasons. These checks are applicable to any one moment distribution and to the sum of any proportions thereof. The rotation checks will expose also any inconsistency in distribution factors within each moment distribution.

(3) Fixed end moments due to any loading can be checked by the requirement, that the free moment area must equal the area formed by the bar and the fixed-end moments. This, however, is not conclusive unless the center of gravity for both these areas lie on the same perpendicular to the bar.

(4) The arbitrary chosen pair of sway moments on a member will cause certain end moments on the other bars, consistent with the geometry of each side sway mode. The ratios for a set of such compatible moments, which should be worked out with great care, will depend on the ratios of respective deflections, stiffnesses and inversely on the ratios of lengths. They can be determined with the use of a Williot diagram. A check can be made by regarding the pin jointed structure as a mechanism, for which the instantaneous angular velocities for each bar are determined using the concept of instantaneous centre. The set of compatible sway moments then equals the set of products of instantaneous angular velocity times stiffness for each bar (see note to point 4)

(5) The establishment of the shears from end moments does not provide an immediate check and should be done with great care. A mistake in these simple steps could not be discovered until the very end of the computations.

(6) Each Maxwell diagram provides a check in itself. In cases of "side sway" and "no side sway", all forces on joints appear in pairs (couples). The reactions, and the shears and axial forces at fixation points should add up to zero.

In the case of the Maxwell diagram for "given loads on joints", these together with all reactions should add up to zero. These checks are necessary but not conclusive.

(7) The equations for K_1, K_2 etc. will give consistent values for K_1, K_2 etc., even if an error should have occurred in (5). The K values might therefore still be incorrect, even if the axial forces established from Eq. (4a) cause the imaginary reactions to vanish. Such errors will however be evident from the final Maxwell diagram.

(8) If an error should have occurred in (5) or in the superposition of moments according to Eq. (3a) or in the subsequent shear determination from final end moments, the final Maxwell diagram will not be consistent, i.e. there will result reactions not equal to zero at the imaginary supports.

If, on the other hand, such reactions vanish in the final Maxwell plan, or if they are negligible, the computations are correct or near correct to such degree of incorrections, as the deviations are different from zero. This is true provided the requirements as stated in points 1 to 4 incl. were met.

(Note to point 4)

Sway Moments for Bent ABCD (Fig. AP1) Are To Be Determined

A compatible set of sway moments can be obtained without displacing any joint, using the concept of instantaneous centre. A compatible and possible set of velocities is given to the joints, the frame being looked upon as pin jointed (see Fig. AP1). Velocity of B is assumed; it is possible only in the direction shown. Velocity of C is then in direction L to CD and as the inst. centre (I.C.) for BC is at X, its magnitude is $v_b \cdot \frac{c}{b}$.

As v_b is the inst. velocity of B, its displacement in time dt would be ds_b and that of C would be ds_c.

The moments set up in a bar fixed at each end when their relative lateral position is changed by ds is given by (see Fig. AP2)

$$M = 6\,\frac{k}{l}\,ds\,E$$

The ratio of any two such Moments

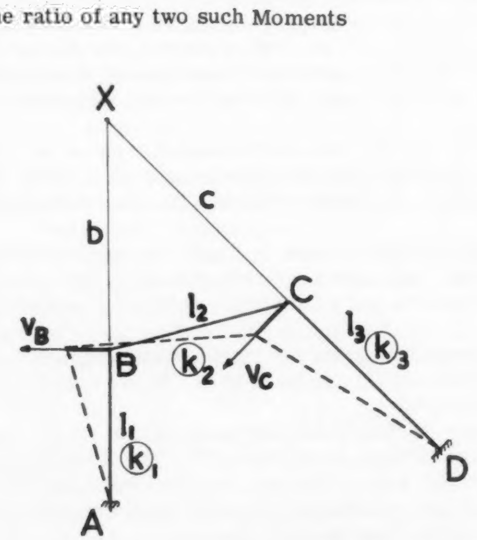


FIG. AP1

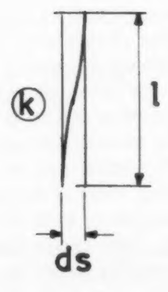


FIG. AP2

$$\frac{M_{BA}}{M_{BC}} = \frac{k_{BA}/l_{BA}\cdot ds_B}{k_{BC}/l_{BC}\cdot ds_{C}}_{\perp BC} = \frac{k_{BA}/l_{BA}\cdot v_B}{k_{BC}/l_{BC}\cdot v_{C}}_{\text{to}B} = \frac{k_{BA}\cdot\omega_{AB}}{k_{BC}\cdot\omega_{BC}}$$

i.e. The set of compatible moments for a sidesway (2 equal moments for each bar) will numerically be equal to a set of the products of instantaneous angular velocity multiplied by the stiffness for each bar.

In the above example if v_b is set unity we have:

$$M_{AB} : M_{BC} : M_{CD} = k_{AB}\cdot\omega_{AB} : k_{BC}\cdot\omega_{BC} : k_{CD}\cdot\omega_{CD}$$

$$= k_1\cdot\frac{1}{l_1} \quad : \quad k_2\cdot\frac{1}{b} \quad : \quad k_3\cdot\frac{c}{b\,l_3}$$

APPENDIX B

Advantage of Use of Maxwell Diagrams

The advantage of using Maxwell diagrams can be appreciated, if it is realized that the Maxwell plan is in fact a solution of 2j simultaneous linear equations, in which the 2j unknowns are: 2j - n axial forces $A_1 \ldots\ldots A_{2j-n}$ and n reactions $R_1 \ldots\ldots R_n$

The equations would be of the form

$$\left.\begin{array}{l} k_{11}A_1 + k_{21}A_2 + k_{31}A_3 + k_{41}A_4 + \cdots = k_{01} \\ k_{12}A_1 + k_{22}A_2 + k_{32}A_3 + k_{42}A_4 + k_{52}A_5 + \cdots = k_{02} \end{array}\right\} \text{joint G}$$

$$\left.\begin{array}{l} k_{13}A_1 + k_{23}A_2 + k_{33}A_3 + k_{43}A_4 + k_{53}A_5 + \cdots = k_{03} \\ k_{14}A_1 + \cdots \qquad \cdots + k_{54}A_5 + \cdots = k_{04} \end{array}\right\} \text{joint C}$$

$$\left.\begin{array}{l} \cdots \qquad \cdots + k_{45}A_4 + k_{55}A_5 + \cdots = k_{05} \\ \text{etc.} \end{array}\right\} \text{joint B}$$

where any of the K values on the left side of the equations (say K_{32}) are trigonometric functions of the angles of members meeting at a joint, and where the K values on the right hand side represent components of forces (including shears) acting at the joints.

For the frame shown in Fig. 7b, some of the K values, i.e. $K_{12} = K_{31} = K_{41}$ etc. would be zero, $K_{11} = 1$, $K_{21} = \cos\alpha$, $K_{22} = \sin\alpha$(considering joint G), while for joint C

$$K_{13} = K_{14} = K_{55} = 0 \qquad K_{24} = -\sin\alpha, \quad K_{34} = +\sin\alpha$$

$$K_{23} = -\cos\alpha \qquad K_{33} = +\cos\alpha \qquad K_{54} = -1 \quad \text{etc.}$$

$$K_{01} = R_{GX}, \qquad K_{02} = R_{GY} \qquad \text{acting at joint G}$$

$$K_{03} = R_{EX} \qquad K_{04} = R_{EY} \qquad \text{" " " C}$$

Thus the system of linear equations would in case of the frame shown in Fig. 7b, reduce to

$$\left.\begin{array}{l} A_1 + \cos\alpha\, A_2 = R_{GX} \\ 0 + \sin\alpha\, A_2 + 0 + 0 = R_{GY} \\ 0 - \cos\alpha\, A_2 + \cos\alpha\, A_3 + 0 + 0 = R_{CX} \\ 0 - \sin\alpha\, A_2 + \sin\alpha\, A_3 + 0 - A_5 = R_{CY} \\ -A_1 + 0 + 0 - A_4 + 0 + 0 + 0 = R_{BX} \\ 0 + 0 + 0 - 0 + A_5 + 0 - A_7 = R_{BY} \\ \text{etc. (4 more equations)} \end{array}\right\} \begin{array}{l} A_1 = \\ A_2 = \\ A_3 = \\ \vdots \\ A_8 = \\ R_1 = \\ R_2 = \end{array}$$

where R_{GX}, R_{GY} etc. are the external forces in X and Y respectively at joint G. R_1 and R_2 are the imaginary reactions at D and E.

If the algebraic procedure above, involving the solution of ten equations is compared with any of the simple line configurations as shown in the illustrations of these pages, the advantage of the use of Maxwell diagrams is evident.

APPENDIX C

Proof of Equations (4)

For any one bar say BC Fig. 5 the total axial force will be:

$$\begin{aligned} A_{BC} = a_{BC} &+ C_1\left[\frac{m_{CD} + m_{DC}}{l_{CD}}\right] + C_2\left[\frac{m_{CE} + m_{EC}}{l_{CE}}\right] + \cdots \\ &+ C_1\left[\frac{m'_{CD} + m'_{DC}}{l_{CD}}\right] k_1 + C_2\left[\frac{m'_{CE} + m'_{EC}}{l_{CE}}\right] k_1 + \cdots \\ &+ C_1\left[\frac{m''_{CD} + m''_{DC}}{l_{CD}}\right] k_2 + C_2\left[\frac{m''_{CE} + m''_{EC}}{l_{CE}}\right] k_2 + \cdots \\ &+ \cdots\cdots\cdots \quad \cdots\cdots\cdots \end{aligned} \tag{6}$$

where C_1 C_2 C_3 etc. are coefficients taking into account the geometry of the frame at joints considered,

m_{cd} is End Moment from moment distribution due to load (no sway)

m'_{cd} is End Moment from moment distribution due to Sway 1.

m''_{cd} is End Moment from moment distribution due to Sway 2.

The square brackets in the Eq. (6) above represent forces perpendicular to the ends of bars concerned. The two* terms in horizontal braces in each line constitute the axial forces in member BC due to end moments from load, from sway 1, and from sway 2 respectively (reading downwards). The sum of all braced terms constitutes the axial force in member BC due to all distributed end moments.

Each vertical brace constitutes the axial force in member BC due to distributed end moments according to "loads" plus mode 1 sway plus mode 2 sway etc. in members CD and CE respectively. Any one of the lines braced in Eq. (6) say line 2 can be rewritten

$$\left[C_1 \frac{m'_{CD}+m'_{DC}}{l_{CD}} + C_2 \frac{m'_{CE}+m'_{EC}}{l_{CE}}\right] k_1 = c_1 k_1 m'_{BC}$$

because all m'_{ik} are in a linear relationship to m'_{bc}. Similarly braced lines 1 or 3 could be rewritten in the same manner giving:

$$A_{BC} = a_{BC} + c_0 m_{BC} + c_1 k_1 m'_{BC} + c_2 k_2 m''_{BC} + \dots$$

plus further terms if there are more than 2 independent side sway modes. The above equation is identical in form with Eq. (4) given earlier in this paper.

REFERENCES

1. Cross and Morgan, Continuous Frames of Reinforced Concrete, John Wiley & Sons.
2. Z. Bazant, Analysis of Framed Structures, Applied Mechanics Review, March, 1955.
3. S. Butterworth, Structural Analysis by Moment Distribution, Longmans Green & Co.
4. Matheson, The Problem of Sway in Complicated Rigid Frames, Journal Inst. Civil Eng. April, 1945.
5. Parcel & Moorman, Analysis of Statically Indeterminate Structures, John Wiley & Sons.
6. Grinter, Modern Steel Structures, Macmillan Coy, N. Y.
7. Grinter, Automatic Design, Macmillan Coy, N. Y.
8. Sterling Kinney, Indeterminate Structural Analysis, Addison Wesley.

*Assuming 3 members meeting at C.

9. Wang, Statically Indeterminate Structures, McGraw Hill.

10. L. C. Maugh, Statically Indeterminate Structures, Wiley & Sons.

11. R. Gartner, Statically Indeterminate Structures, Concrete Publication Ltd., London.

12. Fisher Cassie, Structural Analysis, Longmans Green & Co.

13. Transactions of the ASCE, Vol. 96 (1932) pages 1-156.

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STRUCTURAL DIVISION
Proceedings of the American Society of Civil Engineers

CORRELATION OF PREDICTED AND OBSERVED SUSPENSION BRIDGE BEHAVIOR

George S. Vincent,[1] M. ASCE and M. IABSE

This paper was the basis for an oral presentation at the Joint ASCE-IABSE Meeting at the New York Convention, October 1958. All Joint Meeting papers published in Proceedings or in CIVIL ENGINEERING will be reprinted in one volume.

SYNOPSIS

Doubts concerning the validity of model tests and analysis as an indication of the behavior to be expected of a suspension bridge in the wind are based on (1) a possible scale effect, and (2) the difference between the natural wind and the wind used in the model tests. The first doubt may be discounted because scale effects in aerodynamic studies are generally related to streamlined flow rather than to the turbulent flow characteristic of wind about a bridge. Test data are presented to demonstrate that turbulent or dynamic effects dominate the wind action on bridges.

As to the second doubt, the best indication of uniformity and other characteristics of natural wind may be obtained by a comparison of the behavior of suspension bridges with their behavior as predicted from model tests. The behavior of the original Tacoma Narrows Bridge provides very good correlation with the predictions from model tests and analysis. Even more striking are the results of similar but more extensive studies of the Golden Gate Bridge presented in this paper.

The failure of the original Tacoma Narrows Bridge under wind action on November 7, 1940, placed the engineers of the Washington Toll Bridge

Note: Discussion open until July 1, 1959. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. Paper 1944 is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 85, No. ST 2, February, 1959.

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Authority under the necessity of determining the cause of that failure and developing a design for a structure that would be immune to such action.

The broad implications of this requirement were recognized and as a result an Advisory Board on the Investigation of Suspension Bridges was organized including a large group of engineers concerned with the design of such structures and others familiar with the applications of aerodynamic analysis and research. On the advice of this Advisory Board, the Bureau of Public Roads joined with the Washington State Toll Bridge Authority and the University of Washington to support and participate in the research.

The attack on the problem was further broadened through arrangements in which the Bureau of Public Roads cooperated with other organizations for the following specific studies: Field observations on the Golden Gate Bridge with the Golden Gate Bridge and Highway District,(1) broad theoretical studies involving the services of the late Dr. Friedrich Bleich with the American Institute of Steel Construction(2) and both analytical studies and model tests directed by the late C. B. McCullough with the Oregon State Highway Department.(2)

These various attacks on the problem were correlated continuously and proved to be mutually supporting, each providing answers or clues to problems arising in others.(3)

The tests at the University of Washington and earlier tests at the California Institute of Technology did indeed reveal the nature of the wind action which caused the failure of that bridge as well as its harmless, though significant, bending oscillations during the several months period of its life.(4) The studies also showed the first design proposed for the new bridge to be susceptible to wind excitation and suggested means by which its weakness might be overcome.(4d)

Essentially the favorable features of the new bridge are; first, the fact that the stiffening member is a truss instead of an aerodynamically less stable plate girder, second, a complete bottom lateral system is used, thereby greatly increasing the torsional stiffness of the structure and third, slots are provided in the roadway which reduce to insignificance the tendency of the wind to excite oscillation of the truss-stiffened structure up to any velocity that might occur.

The research has not revealed any consistent and practical method for insuring stability of girder stiffened bridges in general but the quest for stability of such sections is not regarded as hopeless. In quite a number of tests the introduction of roadway slots increased the oscillation of girder-stiffened sections.(4c)

In all suspension bridges of importance built since the failure of the original Tacoma Narrows Bridge, truss stiffening has been used and the bottom lateral system has been incorporated notwithstanding its inhibition of some of the freedom of movement, displacement and stress relief inherent in suspension bridges without bottom lateral systems. Also, in most of these bridges some form of slotting in the roadway deck has been used.

For most recent structures no research on the specific design has been undertaken. This fact gives rise to an obligation to issue a word of warning. It has been found in tests on numerous models and configurations that the beneficial effects of slots are by no means uniform. Moreover, even the benefits of the increased torsional stiffness obtainable by the use of a bottom lateral system vary somewhat. It is easily conceivable that these beneficial

design factors may some day be used in a bridge with disappointing results unless the practice is followed of testing their effects for individual designs.

There are some who accept only qualitatively the results of this research and are disposed to discount the validity of the analytical and laboratory methods as indications of the behavior to be expected of a particular bridge in the winds to which it will be exposed. These doubts arise from two general considerations; first, there are doubts as to possible scale effects which may render the model indication inapplicable or inadequately applicable to the behavior of the full size prototype and, second, it is pointed out that natural winds vary greatly from the ideal, uniform steady winds of known vertical angle of attack which are used in the laboratory.

These doubts certainly merit the most careful consideration and those who raise them are entitled to all of the evidence that can be presented bearing upon these questions. Fortunately, a considerable amount of such evidence has been revealed in the various research projects that have been completed. It is the purpose of this paper to call attention to this evidence.

Scale Effects

Engineers generally realize that there are problems of scale effect that arise in model investigations of airplane sections. These problems arise from the fact that inertial forces from the wind vary as the square of the velocity while viscous or frictional forces, also acting at the same time, vary as the first power of the velocity. Manifestly, there is no velocity scale other than unity (full scale) that will reproduce, to a common scale, forces which vary, respectively, with both the velocity and the square of the velocity. The Reynolds number which occupies an important place in experimental research in aeronautics expresses in significant terms the ratio of the intertial forces to the viscous forces that act upon a given object in a fluid stream. A large Reynolds number indicates that the viscous forces are insignificant compared to the inertial forces. It has been found by research that on blunt or angular sections regardless of size the viscous forces also are more of less insignificant compared to the inertial forces. In passing it should be noted that, as compared with airplane sections, any bridge presents a blunt aspect to the wind. This is confirmed by a study of photographs of smoke streams passing the two types of sections. The contrast in the path of the wind is striking in such photographs; it is generally smooth across the airfoil and always turbulent about the bridge section.

Evidence of a quantitative nature bearing on the scaling of wind forces has evolved from the research at the University of Washington. As a routine part of the tests made on section models of suspension bridges, measurements are made of the logarithmic decrement, a measure of the decay of an oscillation, and also, of the negative logarithmic decrement which is evidenced when motion is gradually built up by wind action. Trace analysis of the motion yields solutions for the logarithmic decrement in still air or in a wind of any velocity and in both cases the values can be determined for various amplitudes of either vertical or torsional motion.(4e) The importance of this arises from the fact that the logarithmic decrement is closely related to the rate of energy transfer, dissipation in the case of a decaying oscillation, or excitation in the case of an increasing motion caused by wind action. It can be readily shown that for low to moderate damping levels the ratio of the energy change per cycle to the energy at the beginning of the cycle is approximately equal to twice the logarithmic decrement.(3a,4f)

In passing it should be noted that the simplicity of the method developed in this research lies largely in the fact that it makes use of quantitative determinations of this rate of energy transfer. It thus eliminates the necessity of tedious measurement of wind forces acting on all portions of the structure and varying throughout the cycle, and of determining mathematically the energy which these forces contribute or dissipate in a cycle.

As previously intimated the test data afford the means of discriminating between the contributions of the inertial forces and the viscous forces of wind action. In order to understand this it is necessary to note the simple mathematical expressions for the ratio of energy transfer per cycle. This is most easily obtained by dividing the energy change in one cycle by the maximum value of the kinetic energy of vibration. The kinetic energy may be expressed:

$$\text{K.E.} = \frac{m\,\omega^2 \eta^2}{2} \tag{1}$$

in which m is the mass of the oscillating body, ω is the circular frequency and η is the maximum (single) amplitude of oscillation.

Considering now the inertial force of still air on a particular section having the mass, m, which is oscillating vertically at a frequency of ω with a maximum amplitude of η it is noted first, that the wind force will be a function of the square of the velocity and of the area and general shape of the section. Since the maximum value of the velocity of the section is $\omega\eta$ such a force can be expressed as $F(t, A)\omega\eta$. This force varies periodically in its intensity as the body moves through one cycle of oscillation and the total work which it performs in the cycle will be the product of the force, and the distance through which the force acts, in which both the force and the distance can be expressed as functions of η. Thus, the energy change can be expressed:

$$\text{Energy change (inertial forces)} = C_1 \omega^2 \eta^3 \tag{2}$$

in which C_1 is a combined constant embracing the area and the wind pressure per unit velocity per unit area as well as integration over one cycle.

The ratio of energy change to energy of vibration then is:

$$\frac{\text{Energy change}}{\text{Kinetic energy}} = \frac{C_1 \omega^2 \eta^3}{\frac{m\omega^2 \eta^2}{2}} = \frac{C_2 \eta}{m} \tag{3}$$

The important point is that the ratio of energy transfer is directly proportional to the amplitude of oscillation, η.

Now, consider similarly the energy change caused by a force which varies as the first power of the velocity. Such a force is proportional to the instantaneous velocity and therefore to the maximum velocity, $\omega\eta$. The work which it performs in one cycle may be expressed:

$$\text{Energy change (viscous forces)} = C_3\, \omega\, \eta^2 \tag{4}$$

and the ratio of energy change to energy of vibration is:

$$\frac{\text{Energy change}}{\text{Kinetic energy}} = \frac{C_3\, \omega \eta^2}{\frac{m\omega^2 \eta^2}{2}} = \frac{C_4}{m\omega} \tag{5}$$

It should be noted here that the ratio of energy change per cycle is constant with respect to the amplitude. This is recognized as characteristic of

viscous damping in which the logarithmic decrement does not vary with amplitude.

Fig. 1 shows a graph in which the logarithmic decrement in still air, observed on a model of the Tacoma Narrows Bridge, is plotted against the amplitude.(4g) It should be noted that the logarithmic decrement at substantially zero amplitudes is quite small, but that it increases linearly with an increase in the amplitude η. The broken line on the curve represents a quantitative determination of the structural damping inherent in the spring system and the supports of the model. This was determined by tests in which the model was replaced by streamlined cylindrical weights of the same mass. Auxiliary tests showed that the aerodynamic forces on these streamlined weights were negligible and thus all of the damping measured was attributable to structural damping. If the values of structural damping, shown by the broken line are subtracted from the corresponding values indicated by the full line, it will be seen that the aerodynamic damping is nearly zero at low values of η and increases in direct or linear proportion with amplitude. This, in the light of the mathematical values derived immediately above, affords substantial proof that the wind forces acting upon the model in oscillation may be regarded as inertial forces and that the viscous forces may be disregarded. Since this is essentially true for the model and almost perfectly true for the large-sized prototype the matter of scale effect is disposed of; the scale for inertial forces applies.

The second factor of uncertainty, which may be loosely classified as a scale effect, is the matter of structural damping. This question concerns the relation between the structural damping of the bridge in the field and the structural damping of the model as tested in the laboratory.

The structural damping of the model and its supports is readily measured as a routine matter, in all tests, as briefly referred to above. The uncertainty lies in the appraisal of the structural damping of the bridge in the field. It is regarded by those who have studied the problem that no model tests will be of any value in indicating the structural damping of the prototype. The various sources of energy absorption due to friction in joints, to contacts between concrete and steel, and to various other factors which might be mentioned cannot be adequately and reliably reproduced in a model. The only source of such information consists of test data on actual suspension bridges. Such test data have been obtained. Arne Selberg in Norway has made a number of tests on bridges of rather moderate spans and has reported logarithmic decrements varying from about .10 to .15 or even greater.(5) Tests have been made and reported in this country indicating values similar to these and somewhat lower.(4h) Further than this, the extensive behavior studies of such large bridges as the original Tacoma Narrows Bridge and the Golden Gate Bridge, when examined in connection with the model tests have yielded fairly good indications, as to what may be the value of the damping on these large structures. It seems quite clear that the damping is much less on large structures than on the smaller ones. This is to be expected when it is considered that the sources of damping in deck contacts and various other elements are more or less equivalent on large and small structures whereas the total energy of oscillation is much greater on the heavier, longer structures which involve the play of tremendous forces in the cables and trusses. Further reference will be made to quantitative data on this factor.

The magnitude of the structural damping of a bridge plays an important part in determining the response of that bridge to the excitation by the wind

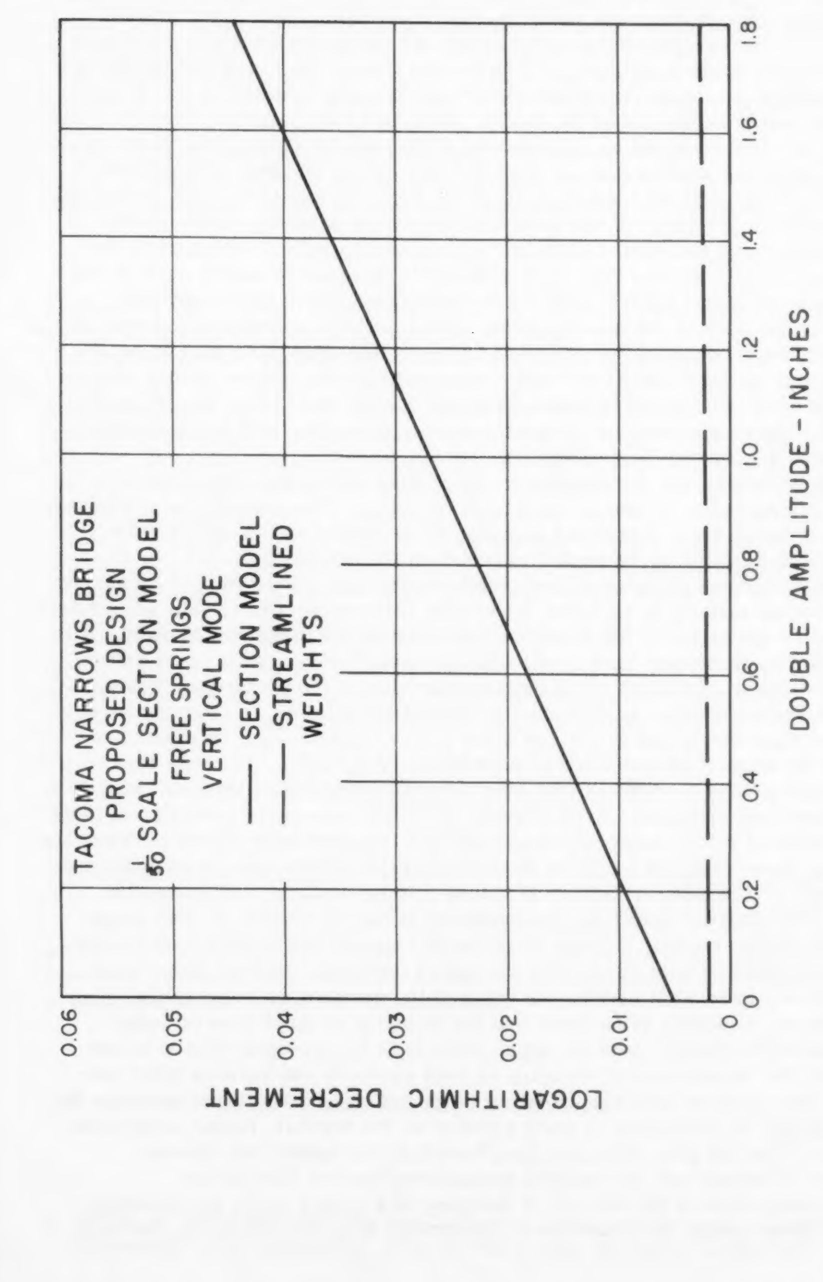

FIG. 1. LOGARITHMIC DECREMENT IN STILL AIR

for some sections, while for other sections its effect is not so great. The routine tests show the effect of structural damping upon the response of a given design to wind excitation. In using the method of analysis which has been developed it is necessary to make an assumption as to the magnitude of the structural damping of the bridge under study. Obviously, in order to be conservative, the assumed damping should be relatively low.

Bridge Behavior

Of course, the real proof of the validity of this method of testing and analysis lies in a comparison of predicted behavior with actual observed behavior of a bridge. The foregoing considerations of scale effects are preliminary in nature but do dispose of one question; they indicate that scale effects do not constitute an inherent barrier to the adequate determination of the behavior of bridges by means of model tests and associated theoretical analysis. The question of correlation, then, revolves on the difference in the nature of the wind in the field as compared to that in the laboratory and uncertainty as to the value of the structural damping of the prototype. As previously suggested, the latter can be estimated with some reliability and conservative values can be used for purposes of analysis. Future research may give more definite information on the actual damping of major suspension bridges. Damping will vary materially with the mode of the motion of the structure, which determines the relative participation of the cable and the stiffening structure in the storage of energy.[2a,4i] This relation has been studied extensively but space permits little discussion here.

Manifestly, the natural wind occurring at any bridge site differs in varying degrees from the uniform, steady wind velocity used in the laboratory and found to be most conducive to the excitation of bridge motion. A wind approaching over an extensive body of water is usually more steady and more uniform over the entire bridge site than a wind approaching over land. Local topographic or artificial features near the bridge modify the wind stream materially. Usually, but not always,[6] the modification involves increased turbulence, reduced uniformity and reduced capacity to excite periodic oscillation of the bridge.

Granting the variability of the wind velocity at a given bridge site, its effect in reducing the potential oscillation of the structure depends very much upon the cross-section of the bridge and the resultant path of the wind stream. There is a substantially linear relation between the wind velocity and the frequency of the oscillation which it is capable of causing and a given mode of oscillation may occur over only a narrow range of velocity. Thus, the original Tacoma Narrows Bridge and its full model in the wind tunnel showed some 9 different vertical bending modes in wind velocities from one to twelve miles per hour (full prototype scale). As a result of normal fluctuations in velocity these vertical modes of motion (and their frequencies) were often mixed as they changed from one to another so that no mode developed to the amplitude which it could have attained in a steady wind.[4a,4c]

On the other hand, the single-noded torsional oscillation which destroyed the bridge could have developed in any wind above about twenty miles per hour were it not for the center cable ties which inhibited this motion until the velocity was so high and forces so great as to overcome their restraint and cause partial failure of the ties. An oscillation which is excited over a wide range of velocity is likely to appear prominently on a bridge even though the

velocity may vary and be non-uniform over the site, provided there is not excessive turbulence or local deviation in the wind stream.

Girder-Stiffened Sections

The full line in Fig. 2 shows the maximum steady-state amplitude for the single-noded torsional oscillation of the original Tacoma Narrows Bridge at a given wind velocity as determined by the model tests(4j) and converted to prototype scale for the frequency of 14 cycles per minute observed on the bridge in this mode on November 7, 1940. At any velocity to the right of the intersection of this curve with the horizontal axis the amplitude will build up to the ordinate of the curve. When, in the model tests, the amplitude was increased by hand-excitation it always damped down in the wind stream to the value indicated by the curve.

For the safety of the model the tests were carried to only about 12° single amplitude. The broken lines represent alternative extrapolations of the test curve. The circle indicates the approximate velocity and amplitude as witnessed on the actual bridge a few moments after the catastrophic torsional oscillation began on the day it was destroyed. It is reasonable that it should fall below the general trend of the experimental curve because of the greatly increased structural damping induced by inelastic deformation of elements of the bridge at the great amplitude reached.

The observed behavior of the bridge is remarkably consistent with the indications of the model tests which show the great danger from torsional oscillation in winds exceeding 20 miles per hour. It must be remembered, however, that these test data were not available beforehand to give warning of the failure of the bridge.

The inconspicuous curve between seven and ten miles per hour shows a very weak response of this torsional oscillation which was observed on the model, only. Such early appearances, sometimes fairly strong, of both vertical and torsional modes are often found in model tests and a whole series of successive early responses is theoretically possible.

Fig. 3 shows the results of a statistical analysis of field observations of vertical oscillations of the original Tacoma Narrows Bridge during its brief existence. The diagram is taken from the report to John M. Carmody, Administrator of the Federal Works Agency, by the Board of Engineers consisting of Othmar H. Ammann, Theodore von Karman and Glenn B. Woodruff (reproduced in part in reference 4a). The plotted velocities have been corrected in accordance with the subsequent calibration of the anemometer as shown in Fig. A-3 of reference 4a. In this figure each area represents an observed vertical mode of oscillation of the bridge. Its base is plotted on the ordinate representing the observed frequency of oscillation of the mode. The vertical dimensions of each block are proportional to the number of observations showing a particular mode of oscillation at the particular wind velocity shown by the abscissa. A small circle marks the abscissa of the center of gravity of each area.

The strong trend toward a linear relation between the wind velocity and the frequency of the oscillation which the wind can excite is evident in these diagrams. This relation is conveniently expressed by the ratio, V/nb, in which V is the velocity in feet per second, n is the frequency in oscillations per second and b is the width of the section in feet. Being dimensionless, it is equally applicable to a bridge and to any scale model of it. This is perhaps

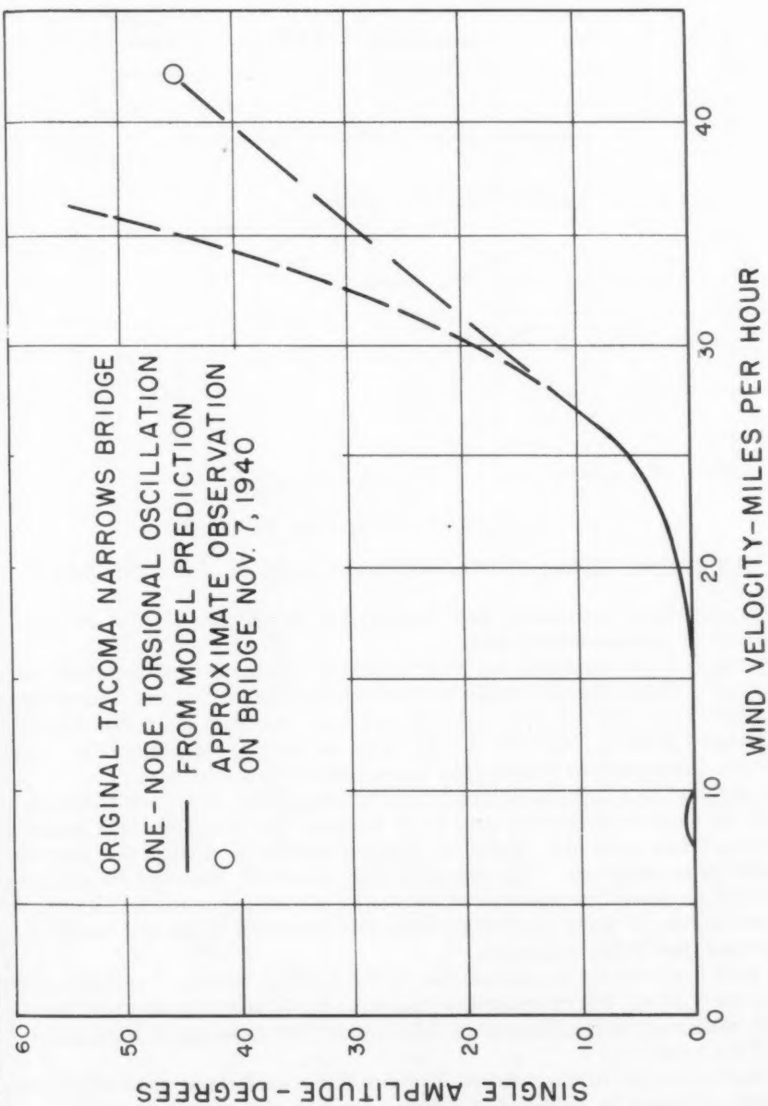

FIG. 2 TORSIONAL OSCILLATION - ORIGINAL TACOMA NARROWS BRIDGE

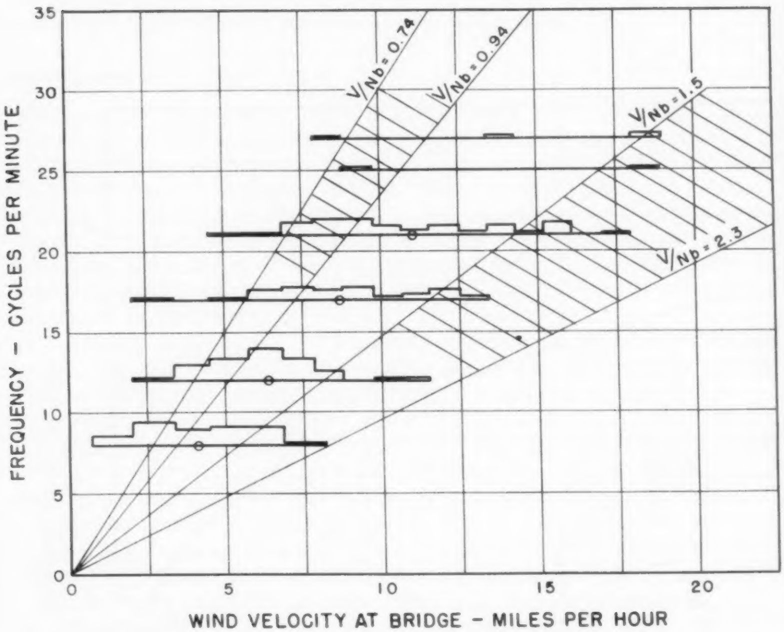

FIG. 3 VERTICAL OSCILLATIONS - ORIGINAL TACOMA NARROWS BRIDGE

the most prominent parameter which appears in the studies of wind excited oscillations of suspension bridges.

The diagonal lines indicate the V/nb ratios at which vertical oscillations were initiated in the section model tests at California Institute of Technology. The shaded range between 0.74 and 0.94 was for weak first appearances and that between 1.5 and 2.3 was for the stronger second appearances. The model tests of the University of Washington fall within the second range.

The fact that so many observations on the bridge fall within the indicated range of the weak first appearances must indicate that the structural damping of the bridge was quite low. Some oscillations occurred in winds too light to be sensed by an observer. The observed data, however, must not be accepted uncritically as quantitive because of the material variations in velocity which must have occurred along the bridge from that recorded at the toll house at the southeast end of the structure.

For such behavior as is exemplified by the several modes of vertical oscillation of the Tacoma Narrows Bridge the variation of the natural wind from the ideal wind used in the laboratory does reduce the severity of wind excitation of the bridge.

The section model of the original Tacoma Narrows Bridge was tested with its girders replaced by a series of girders representing prototype depths from 5.4 ft. to 11.5 ft.(4k) The last had a ratio of girder depth to bridge width slightly greater than that of the Deer Isle Bridge and was, by coincidence, a fair 30-scale model of that bridge except that the narrow slots between the roadway and the girder webs were lacking. Fig. 4 shows by full lines the

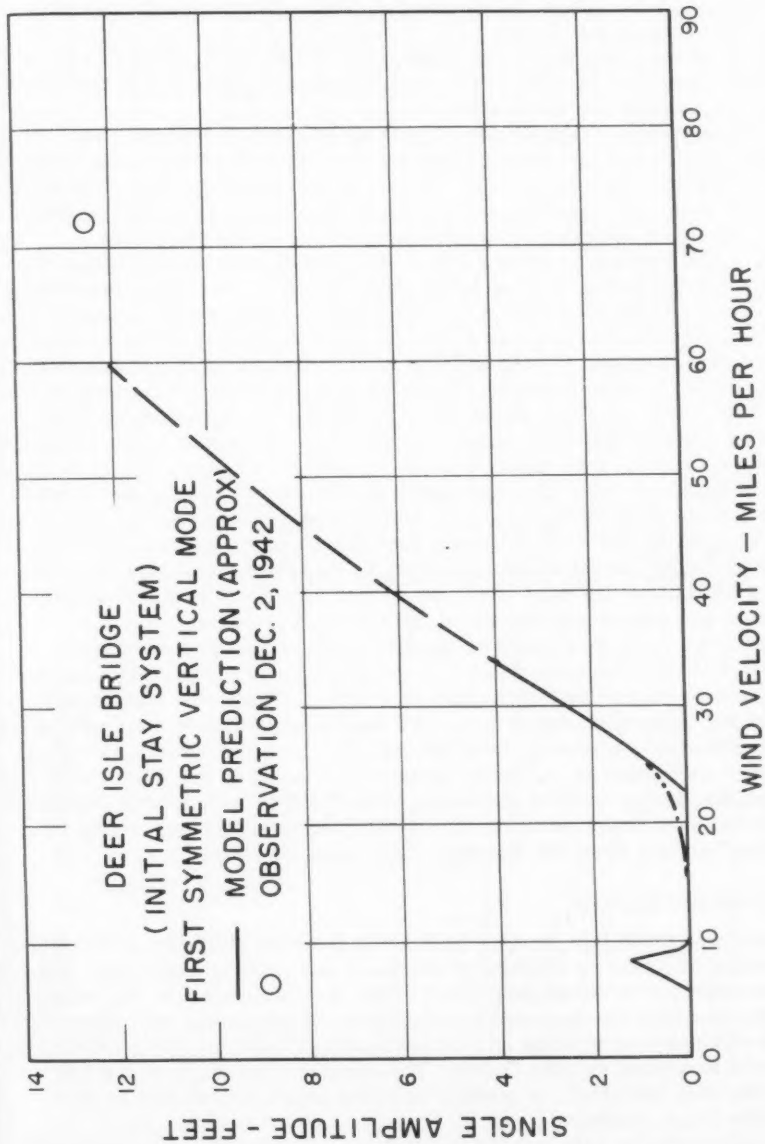

FIG. 4 VERTICAL OSCILLATION - DEER ISLE BRIDGE

response of this model converted to full prototype scale for the Deer Isle Bridge. It is seen that a reasonable extrapolation of this curve is entirely consistent with the one published observation of wind velocity and amplitude of oscillation in the storm of December 2, 1942. This was described as a single loop vertical oscillation.(7)

Fig. 4 shows also a weak first appearance of this mode which developed in the model tests and, by dotted line, a weak response which just preceded the strong oscillations discussed above.

Tests on models of the Bronx-Whitestone Bridge,(41) both with and without the added truss, indicate strong torsional oscillation in uniform steady winds of moderate velocity normal to the bridge. Such oscillation has never been observed on the actual bridge and it is concluded that at this site the natural wind must surely lack the uniformity necessary to effect excitation.

The model test also showed small amplitudes of vertical oscillation at a velocity corresponding to about fifteen miles per hour on the prototype but when the curves for logarithmic decrement were developed it was apparent that their negative damping was so low as to be completely offset by a moderate structural damping even in an ideal wind so that no prototype response to this excitation is to be expected. Nevertheless, the bridge in its original condition before the truss was added did show sizeable oscillations, up to about 30 inches double amplitudes, in an assymetric vertical mode. This occurred only in a quartering wind and is believed to have been caused, not by resonant action of the wind, but by the momentary displacement of the bridge longitudinally in response to wind gust action, and the consequent swaying of the bridge in the longitudinal direction following such displacement. It has been observed on the full model at the University of Washington that an assymetric vertical oscillation can most easily be excited by a slight longitudinal displacement and subsequent release of the suspended structure.

Tests were made in England on the full model of the proposed Severn Bridge(8,9,10) to determine whether this oscillation in the first asymmetric mode could be excited by longitudinal gust action. The model was rotated on its turntable support to expose it to a 45° quartering wind and a venetian blind type of shutter was placed in the wind upstream from the model. All efforts to produce oscillation by rhythmic opening and closing of the shutter failed, but when the shutter was left stationary in the wind an asymmetric oscillation such as had been observed on the Bronx-Whitestone Bridge was set up by "buffeting" arising from the disturbed flow about the shutter.

Truss-Stiffened Sections

All tests on truss type bridges reveal that the wind excitation of these is a modification of flutter, a coupling of torsional and vertical oscillation, which is so troublesome in the airplane field. With the blunt sections of a bridge it is greatly modified and somewhat ameliorated but associated with other exciting forces which tend to bring on less serious oscillation at lower velocities than would be caused by pure flutter. The excitation arises from the flow around the deck structure. A positive damping effect is produced by flow around the truss members.

It is interesting to note that the original record of the damage sustained by the Menai Straits Bridge in 1836(4m) reveals, in the light of modern research, that flutter was the cause of the excitation of that structure. The leeward side suffered much greater damage than did the windward side of the bridge which

must indicate that it underwent greater distortion. In flutter, combining both vertical and torsional movement the leeward edge moves with greater amplitude than does the windward edge.

Section models of the George Washington Bridge with the middle portion of the deck open as originally constructed and also with it decked over as at present were tested in the wind tunnel.(41) In the open condition small vertical and torsional oscillations developed at velocities corresponding to 12-16 and 18-24 miles per hour on the prototype in the first symmetric mode (three loops) and at about 75% of these velocities in the first asymmetric mode (two loops). However, the logarithmic decrement curves showed maximum negative damping values of only 0.03 and 0.011, too weak to overcome the structural damping so that no oscillation of the prototype could be expected. Movements of the bridge itself were too slight to be identified as to mode.

With the entire deck closed the above-mentioned weak oscillations did not appear in the model but a strong torsional oscillation developed at a velocity corresponding to 65 miles per hour computed for the symmetric mode and 50 miles per hour computed for the asymmetric mode of the bridge itself. These model tests were not extended to evaluate the influence of structural damping on the critical velocity (which <u>is</u> indicated in Fig. 5 for the Golden Gate Bridge) but the structural damping of the bridge would undoubtedly raise the critical velocities well above the values stated above. Dr. Friedrich Bleich's flutter analysis utilizing model test data and an assumed value of structural damping indicates a critical velocity of 77 miles per hour for flutter.

In view of the rarity of high winds and the probability of factors tending to produce non-uniformity in the wind stream it is not surprising that the torsional oscillation has not been observed on the George Washington bridge.

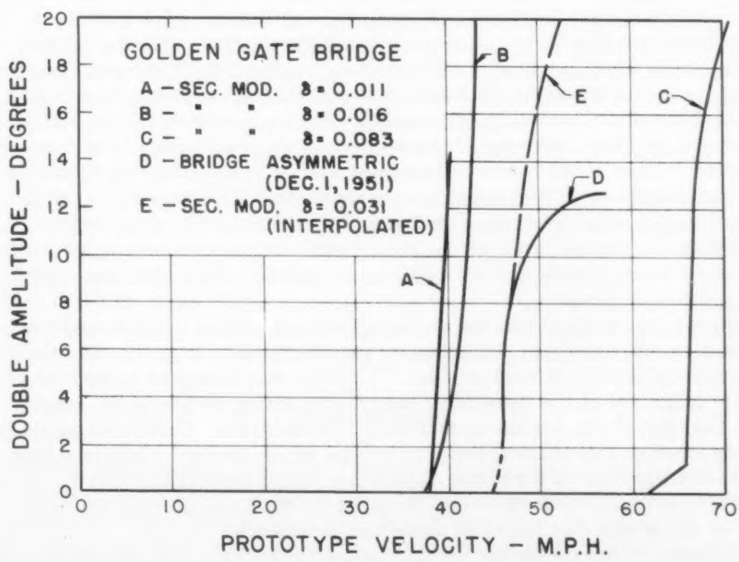

FIG. 5 TORSIONAL OSCILLATION OF GOLDEN GATE BRIDGE

The studies on the Golden Gate Bridge represent perhaps the most interesting and convincing evidence on this question of the correlation of laboratory indications and actual bridge behavior.(1) The field observations are more complete than on any other bridge. Over a period of several years complete records of the motions of the bridge at ten different points on the structure were made and these were correlated continuously with the wind velocity and direction as observed at a point several feet above and west of the west cable at the middle of the main span. Because of the nature of the approach here with the prevailing westerly wind off the ocean, the wind through the Golden Gate is probably as uniform as can be expected in any location. However, for a period of fifteen years the bridge response was trivial as compared with the predictions made from the combined model studies and theoretical analyses. The reasons for this were readily found when a search was made for disturbing influences. The north sidespan is located just east of a steep hill which reaches a height of nearly 1,000 ft. As a result the wind on this side span has a very pronounced downward component. Laboratory tests on a full model of the new Tacoma Narrows Bridge in a configuration subject to excitation, showed that changing the wind to a sharp downward angle on one side span would break up the symmetric oscillation previously induced by a uniform horizontal wind. The principal so clearly shown in the model tests on the Narrows bridge accounts for the small amplitudes of the symmetric modes of oscillation of the Golden Gate bridge over the fifteen-year period mentioned.

Verification of the model tests and rational analysis was obtained on the only known occasion when the bridge responded in the first asymmetric mode which requires no movement of the side spans and is not responsive to forces acting upon the side spans. The record of this is shown in Fig. 5.

The motion on December 1, 1951, which is plotted as curve D on this graph, was a combination of vertical and torsional oscillation. The graph indicates, for convenience, the torsional component in comparison with the model indications which also are plotted. The values plotted from the model tests in curves A, B and C are, in all cases, for a horizontal wind but at different values of the structural damping as shown by the values of the logarithmic decrement, δ. The influence of the structural damping on the critical velocity is apparent from these curves. It is interesting to note that the behavior of the actual bridge correlates remarkably well with an interpolated curve for a value of δ of 0.031. The curve representing actual bridge amplitude falls below the model curve at the higher velocities, because the higher velocities never persisted in the field long enough to induce the amplitudes that were inherently possible.

It should be recalled that the Golden Gate Bridge has been strengthened against a reoccurrence of the behavior represented by Fig. 5 by the addition of a complete bottom lateral system.(11) This was designed in accordance with the judgment of the consulting board, consisting of Mr. C. E. Paine, Chairman; Mr. O. H. Ammann, and Mr. C. E. Andrew. Tests and analysis of the type employed in making the predictions shown in Fig. 5 indicate that the increased frequency of torsional oscillation which results directly from the use of the bottom lateral system will increase the critical velocity beyond the range of any winds that might be expected at this site.

All phases of bridge design involve assumptions as to the nature and magnitude of the loading to be applied, whether they are live loads or natural forces. The greater uncertainty is usually involved in assumptions concerning such

natural forces as wind pressure, flood height, etc. It can rarely be said, for example, that the floods and scouring conditions that actually occur during the life of a structure even approximate the assumptions made in the design. Nevertheless, such assumptions must be made in order to proceed with the design.

The model tests and analysis showed the new Tacoma Narrows bridge to be stable. The bridge has shown no trace of oscillation during the eight years since it was built. The same type of model tests and analysis confirmed the stability of the Mackinac Straits Bridge.

The aerodynamic behavior of suspension bridges in actual service has been predicted and reproduced so successfully and consistently in model tests that it would seem to the writer, highly imprudent not to make full use of such tests in the design of every important suspension bridge. Failure to attribute full validity to a type of investigation and analysis that has so impressive a record of correlation and applicability, and failure to use it, involves too great a risk and too high a probability of damage to or even loss of the structure.

The opinion has been expressed that a large bridge because of its great mass will inherently benefit from increased damping and, therefore, offer increased resistance to excitation by the wind. It is not true that the large structure has greater damping but it is acknowledged that, because of the great mass, its energy of oscillation is high and, therefore, the energy of excitation must be correspondingly great before motions of any magnitude or significance can be induced. This is shown in the derivation of Eqs. (3) and (5) which have the mass, m, in their denominators. Furthermore, it may be presumed that the opportunities for variation in the wind velocity, direction and behavior are greater over a long bridge than over a short one. Detracting from these advantages, however, is the fact that the oscillation of any long bridge will have a lower frequency than that of a short bridge. This is true despite anything that can be done to stiffen it. It is inherent in the fact that the span is long. Research has shown indisputably that the relation between wind velocity and the frequencies of the motion which it may induce is a definite factor. Therefore, a long bridge having a low natural frequency, can be excited by a lower wind velocity than can a bridge of similar cross section, of shorter span and higher frequency.

It is the judgment of the writer that the aerodynamic characteristics of the design of any bridge of considerable magnitude should be investigated by the means now available.[4,12,13] This investigation will predict the behavior of the bridge in a full range of wind conditions. The predictions take fully into account both the mass of the structure and its stiffness as these factors affect the frequency of oscillation in any possible mode and the amount of energy required to build up a given amplitude of displacement.

REFERENCES

1. "Golden Gate Bridge Vibration Studies"—A report on Cooperative Research Project conducted by the Golden Gate Bridge and Highway District and the Bureau of Public Roads, Department of Commerce, by George S. Vincent, Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 84, ST6, October, 1958.

2. "The Mathematical Theory of Vibration in Suspension Bridges", by Friedrich Bleich, C. B. McCullough, Richard Rosecrans and George S. Vincent, Government Printing Office.

   (a) Chapter III, Part 4.

3. "Aerodynamic Stability of Suspension Bridges", 1952 Report of the Advisory Board on the Investigation of Suspension Bridges, Transactions, American Society of Civil Engineers, Vol. 120 (1955) page 721.

4. "Aerodynamic Stability of Suspension Bridges with Special Reference to the Tacoma Narrows Bridge", University of Washington Engineering Experiment Station Bulletin No. 116:

   (a) Part I, "Investigations Prior to October, 1941" by F. B. Farquharson.
   (b) Part II, "Mathematical Analyses" by Frederick C. Smith and George S. Vincent.
   (c) Part III, "The Investigation of Models of the Original Tacoma Narrows Bridge Under the Action of Wind", by F. B. Farquharson.
   (d) Part IV, "The Investigation of Models of the New Tacoma Narrows Bridge Under the Action of Wind", by F. B. Farquharson.
   (e) Part V, "Extended Studies: Logarithmic Decrement, Field Damping, Prototype Predictions, Four Other Bridges", by George S. Vincent.
   (f) Part V, Appendix I.
   (g) Part V, Chapter I.
   (h) Part V, Chapter II.
   (i) Part II, Chapter VII.
   (j) Part III, Fig. 41.
   (k) Part III, Fig. 116.
   (l) Part V, Chapter IV.
   (m) Part I, Appendix I.

5. "Dampening Effect in Suspension Bridges", by Arne Selberg. Publications, International Association for Bridge and Structural Engineering, Vol. X (1950).

6. "Aerodynamic Buffeting of Bridges" by C. Scruton, The Engineer, (Great Britain) May 13, 1955.

7. "Rigidity and Aerodynamic Stability of Suspension Bridges", by D. B. Steinman, Transactions American Society of Civil Engineers, Vol. 110 1945), page 492 (See page 544).

8. "A Summarized Account of the Severn Bridge Aerodynamic Investigations" by R. A. Frazer and C. Scruton, H.M.S.O. 1952.

9. "An Experimental Investigation of the Aerodynamic Stability of Suspension Bridges, with Special Reference to the Proposed Severn Bridge" by C. Scruton, Proceedings, Institute of Civil Engineers, 1952, Part 1, Vol. I.

10. "An Investigation of the Oscillations of Suspension Bridges in Wind", by C. Scruton, Fourth Congress of the International Association for Bridge and Structural Engineering, Preliminary Publication, 1952.

11. "Golden Gate Bridge is Stiffened Against Wind Vibration", Engineering News-Record, Vol. 152, (June, 1954).

12. "Mathematical Prediction of Suspension Bridge Behavior in Wind from Dynamic Section Model Tests" by George S. Vincent. Publications, International Association for Bridge and Structural Engineering, Vol. XII, (1952).

13. "Aerodynamic Characteristics of a Suspension Bridge", by George S. Vincent, Public Roads, Vol. 28, No. 8 (June, 1955).

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CONTENTS

DISCUSSION

(Over)

*There will be no closure

SYNOPSIS OF FIRST PROGRESS REPORT OF COMMITTEE ON FACTORS OF SAFETY[a]

Discussions by R. J. Ruble and Raymond Archibald
Closure by Oliver G. Julian and Alfred M. Freudenthal

E. J. RUBLE,[1] M. ASCE.—The progress report of the Committee on Factors of Safety is certainly an outstanding paper and should be of considerable help to engineers studying the subject of probability of occurrences. However, it is very doubtful, in the writer's opinion, that the method of attack covered in the paper will ever be accepted by specification writing bodies to establish definite factors for design purposes, especially for structures subjected to moving loads.

There are many practical factors which must be considered by a committee before any reliable factors of safety can be established. The bridge engineers responsible for the design and maintenance of structures carrying moving loads and where danger to human lives is involved are going to base their recommendations on past experiences. The writer has had considerable experience working with various specification writing bodies, both railway and highway, and these committees always consider the following factors which have been determined by laboratory investigations and research on structures under actual operating conditions:

Static Loads

1. From the railroad standpoint, the committees realize that there is considerable variation in the static weights between the wheels and axles in the same locomotive. It has been determined by modern scales which weigh each wheel that there can be a 25 percent variation in axle weights from one weighing to the next weighing. In other words, a locomotive can be run on these scales and each wheel weighed. The locomotive can be then run off the scale and immediately run it on the scale again and there will be at least a 25 percent variation in the weights of the axles. This is nothing unusual and undoubtedly results from friction in the journal boxes and in the equalizing systems. In a similar manner there will be a variation between the weights of the two wheels on the same axle. There is no doubt but what the weights of the axles and wheels on some of the large highway trucks will vary in a like manner. It would certainly be interesting to see some data on this particular item.

2. It is well known that there is considerable variation in the axle weights and total weights between locomotives of the same class and built at the same time.

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a. Proc. Paper 1316, July, 1957, by Oliver G. Julian.
1. Research Engr. Structures, Association of American Railroads, Chicago, Ill.

3. The amount of fuel varies from one division point to the next so that the total static weight of the locomotive will vary depending upon where the bridge is located.

4. There is considerable variation in the weights of the various cars following the locomotive. The railroads have special cars at the present time passing over the bridges that produce static bending moments twice as great as those of our standard design loading of Cooper's E72.

5. The committees always take into consideration the possibility of future loads. Most of the railroad bridges in this country for many years were carrying loads about 50 percent greater than their design loads. The majority of the bridges were built at the beginning of the century and were designed for about an E45 loading. The weights of the locomotives gradually increased until they finally reached about E70. The advent of the diesel locomotives reduced these loads to about their design loads. The bending moments produced by future atomic locomotives and the gas turbines should be considered as well as the possibility of special cars even heavier than those we now have.

6. The possibility of increased dead load on the structure is usually considered. The railroads have a large number of bridges where the dead load has been increased at least two or three times over the design dead load. This came about by designing the bridge for an open timber deck. However, many years later the railroad decided to place a ballasted deck on the structure. Their argument was that the ballasted deck reduced the impact which compensated for the increased dead load and some data on this subject supports this argument.

Dynamic Loads

1. The dynamic effect of a moving vehicle, such as a train or truck with a large part of its weight supported on springs, moving across a bridge, which in itself acts as a spring, is a very complicated action and produces effects of various magnitude. It has been determined from extensive field tests that even when trains cross the bridge at the same speed, there is considerable variation in the dynamic effects.

2. It has been determined by strain gage measurements in the field on a large number of structures that many times a negative impact effect is recorded. In other words, the stresses produced by the locomotive at a particular speed are lower than those recorded with the locomotive standing on the bridge or moving slowly across the bridge.

3. The rolling of the locomotive and equipment about a longitudinal axis tends to increase the stresses on one side with a corresponding decrease in stresses on the other side.

4. The rigidity of the supports or the flexibility of the structure plays a very important part in the dynamic effect. The dynamic effect in a bridge resting on rigid supports is considerably greater than the effect in a bridge supported on tall viaduct columns.

Distribution of Loads

1. It has been determined that there is considerable variation in the stresses, both static and dynamic, in bridges consisting of two or three beams per rail. Typical structures of this type are beam spans and stringers in truss spans. For example, where there are two beams or stringers under one rail it has been found that the stresses in one beam or stringer may be considerably greater than the stresses in the other beam or stringer.

2. The committees know that there is considerable variation in the stresses of any member. In a plate girder the stress is not uniform across the section and is usually higher on one edge than on the other edge. In a built up chord member of a truss there is always bending about both axes of these members. Many chord members have tension on one corner of compression members under the passage of moving loads and corresponding compression in one corner of tension members. It is usually considered that small secondary stresses in members are not harmful and this is undoubtedly true for the static strength of tension members. However, these large bending stresses must certainly reduce the buckling strength of compression members and must have a definite influence on the fatigue strength of the members. It has been observed that many individual bars of eye-bar members were not carrying any of the load with a corresponding increase in the other bars of the member. In addition, many bars have zero stress on one edge of pin-connected eye-bars with twice the average stress on the other edge. Another interesting variation of stress has been observed in a viaduct tower. One particular column of the tower which appeared to be raised above the concrete pedestal never came into bearing on the pedestal under the passage of a train. As a result of this unequal distribution the other columns were carrying more than their proportion of the load.

Material - Steel

1. The committees are well acquainted with the variation in the yield point and the ultimate strength of our structural steels. The steel is usually ordered to meet definite requirements in the mill and these requirements are based on certain methods of testing. It has been shown that in many cases the actual yield point of steel is considerably lower than the requirements set up in the specifications due principally to the more refined method of testing in the laboratory.

2. A vast amount of data has been collected on the fatigue strength of structural joints, but the research engineers still cannot say that any particular joint will have a certain fatigue strength at a definite number of cycles of known load. The committees realize it will be a good many years before sufficient data will be available to prepare definite S-N curves.

3. The method of fitting up the steel, driving the rivets and the type of rivets is an important factor in the strength of any joint. It is well known that both good and poor rivets are driven and while the poor rivets are removed, very little is known about what we do to the good rivets when we remove the poor rivets. The character of the rivets also influences the fatigue strength of the joints. It is realized that if the rivets do not have good clamping action there will be some slippage of the plates resulting in high bearing pressure on the edges of the rivet holes with a resulting stress concentration factor of 25 or 30 or even higher. This particular factor undoubtedly accounts for some of the actual fatigue failures in the field which cannot be accounted for by laboratory tests on typical specimens.

4. The fatigue strength of any structure is influenced by the variation in the range of stress. Practically all of the present data on fatigue are based on a constant stress range. In actual structures there is a considerable range in the stresses resulting from different weight locomotives or vehicles and variations in dynamic effects.

5. The capacity of columns has been of interest to the engineering profession for many years and a tremendous amount of data have been collected on the subject. However, very little data are available on the actual buckling strength of such members when they are in actual structures. A column in a structure is subjected to various degrees of end bending moments resulting in bending about both axes of the column.

6. It is realized that most of the heavier rolled sections have residual stresses resulting from the cooling of the member, but very little data is available on the effect of these residual stresses on the carrying capacity of the members.

7. It is very unusual to find a bridge, both railroad and highway, in perfect condition. You will usually find some of the members with bent angles or plates. Sometimes these damaged members result from improper handling during erection, but usually they are the result of misplaced loads or out-of-control vehicles hitting the member. These damaged members certainly have some influence on the ultimate carrying capacity of the structure. In addition to the visible damaged members, very little is known about the damage to the steel resulting from the conditioning clauses in the specifications. In accordance with specifications, the steel mills are permitted to grind out flaws in the steel. Under certain conditions they just smooth out the edges by grinding while in other cases they build up the ground out areas by welding and then grind smooth. It is doubtful that these conditioning clauses have any influence on steel built into structural joints, but they might have some influence on other members not fabricated into joints.

8. Any steel structure that is not properly protected will have some loss in section due to corrosion. This loss due to corrosion can be quite severe in certain structures close to industrial areas or in structures subjected to brine drippings. The railroads have considerable trouble with loss of section due to the brine from refrigerator cars and many top cover plates on bridges have about 90 percent of the section gone. Many times the rivet head is entirely gone and in other cases the webs of floor beams will have large holes in them due to this corrosion.

Materials - Concrete

1. The committees have had many discussions on the variation in cylinder strength and they all know that there is considerable range in these strengths. However, very little discussion has taken place in the committees regarding the reason for this range. It is known that the method of preparing, curing, capping and testing the cylinders plays an important part in their strength. In addition to these factors, the committees usually consider the relative strength of the concrete in the structure as compared with that in the cylinder.

2. Consideration is usually taken of the actual size of the cylinder as compared to the size of the structural member. It is known that a variation exists in the indicated strength of the concrete by varying the size of the cylinders. The committees also consider the available data on the strength of the concrete as indicated by the cylinders as compared to the strength of the concrete determined from tests on cores removed from the structure.

3. It is well known that the same variations exist in the yield point and ultimate strength of reinforcement as found in structural steel. In many structures of reinforced concrete, the yield point of the reinforcement actually determines the strength of the structure. A typical example of such a structure

is reinforced concrete pipe culverts. These culverts usually fail shortly after the yield point of the reinforcement is reached.

4. Considerable trouble has been encountered in the past with the deterioration of concrete, so the committees must take into account how much deterioration can be permitted before the structure will fail.

5. The committees realize their specifications for the design of structures is based on certain assumptions and allowances for these assumptions must be made in their factors of safety. Possibly the most outstanding example of these assumptions in concrete design is that pertaining to shear reinforcement. All laboratory tests definitely show that shear reinforcement in concrete members do not carry any stress until the concrete has practically failed in diagonal tension. However, the assumptions for the design of these members are based on the concrete carrying a certain amount of load and the web reinforcement carrying the remaining load.

6. Another item which is usually considered in determining the factors of safety deals with the method of construction or erection. One example pertains to the installation of reinforced concrete culvert pipe where they were improperly installed and large cracks developed even before the final fill was placed over the pipe.

Materials - Timber

1. The ultimate strength of structural timbers is determined by the total duration of load on the structure. This load does not have to be continuously applied, but small increments of loading time can be added together to give the same duration of time. The present stresses for structural timbers are based on a duration of load of 10 years and the committees will certainly consider a revision of this loading time as research develops more information.

2. The committees realize that under certain conditions untreated timbers will decay, but they have been able to practically eliminate this decay by preservative treatments. However, there is considerable discussion that the heat resulting from the treatment of the timber reduces the ultimate strength of the timber. It is now known that before many years practically all of the timber structures in the United States will be subject to termite attack. The proper treatment and amount of retention of the treatment to resist termite attack is not too well known, so here again an item which must be considered in the factors of safety.

3. It is well known that all structural timber is graded according to certain rules, such as the number and size of the knots and the slope of the grain. Various grades of timber have various structural strengths and there is undoubtedly some variation between different graders resulting in some variation in the strength of the timber.

4. The checking of heavy timbers plays a very important part in the strength of such members in horizontal shear. Specifications permit a certain value for the horizontal shear under certain loading conditions, but the actual strength of the timber will vary with the amount of checking.

5. Considerable data are available on the strength of timber bolted joints when subjected to static loads, but very few tests have been made on such joints when subjected to repeated loads, consequently, the committees must be guided by past experience in arriving at proper design values for such joints. It has now been shown that the strength of such joints are materially reduced when subjected to repeated loads.

6. The committees know that any joint connected with a mechanical anchor and a bolt is considerably increased over the strength when only a bolt of the same size is used. This is true under both static and repeated loading. However, it has been shown in the field that all of the mechanical anchors are not seated the same amount. Here again the committees depend upon the full seating of the anchor and the factors of safety must take care of the variation.

The writer realizes that this is rather a long discussion and many of the factors discussed do not necessarily have to be considered numerically in arriving at a proper factor of safety, but they still enter into the ultimate strength of the structure. It is difficult to see what will be gained by making a statistical study of the variation in the static loads crossing the structure. The committees are interested in the total static plus dynamic load on the structure and the only possible way that this can be determined is by comprehensive tests on actual structures in the field such as the railroads have been conducting for the last fifteen years and the highways for the past several years. With this data, the effect of such a load pattern on the structure can be determined in the laboratory. For example, the Research Council on Riveted and Bolted Structural Joints is studying the effect of a variable stress pattern on structural joints, typical of that occurring on railroad bridges, as determined by extensive field tests.

The railroad committees are not in a position at this time to say what factor of safety should be used in railroad bridges. The present factor results in a design stress of 18,000 psi in structural steel bridges and experience has indicated this factor is too low for such bridge members as floorbeam hangers where the railroads have experienced a large number of fatigue failures. Possibly the factor for longer girder spans and chord members of truss spans could be reduced somewhat, but here again, experience has shown that a stress of 24,000 psi, which is the rating rules stress for the older bridges, is too high as the bridges were beginning to develop fatigue failures before the loads were reduced by the use of the diesel locomotives.

RAYMOND ARCHIBALD,[1] M. ASCE.—The Factors of Safety Committee, organized to delve into the use and misuse of the term by the engineering profession, has covered a large field of study in their deliberations. During this period of study the committee has developed what it considers the proper statement of the facts involved in determining the factors of safety of structures, and has rightfully indulged in the use of the probability of occurrence factors which enter into design. It is noted in the Synopsis of the First Progress Report of the Committee on Factors of Safety that "some members of the committee inclined to the view that the committee is overemphasizing the importance of the statical and probability studies." It is believed by the writer, who is a member of the committee, it could more accurately be stated that some members of the committee do not agree with the application that has been made of statical and probability studies in some cases.

In the progress report the terms "mean estimated resistance" and "mean load effect" are used to define the minimum required factor of safety. It is at this point where the divergence of opinion develops.

The writer believes where danger to human lives is involved in the design of structures such as highway bridges, railroad bridges, and buildings where

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public gatherings are held, that the definition for the minimum required factor of safety should be the ratio of the minimum estimated resistance to collapse during the anticipated life of a large number of structures meant to be identical with the subject structure, and the maximum load effect for which the subject structure is designed. It will be noted that the only difference in the definition herein proposed is that the terms "minimum" and "maximum" are substituted for the term "mean" in the proposed definition. It should also be emphasized that the definition proposed by the writer is suggested for those cases where danger to human life is involved. In other cases the "mean" method certainly should be given consideration.

In the design of structures by the ultimate load method the maximum load that a structure would sustain at or before failure is used for the design load. The ultimate load is usually arrived at through the use of the term live load factor, which is a load equivalent to two, three, or four times a design load using working stresses. It simply means that the structure will sustain the ultimate load used in the design without failure.

As noted in the report, the ultimate capacity is not necessarily the controlling factor in the design in many instances, but the factor of serviceability is the all-important limitation in many cases.

The question is naturally asked why the minimum estimated resistance to collapse is suggested rather than the mean estimated resistance to collapse. Assuming the resistance to collapse is dependent upon the strength of a material, this leads to the assumption that the minimum estimated resistance is logically based on the minimum strength of the material used in the structure. Present day specifications for structural material include, as a rule, a minimum yield point and ultimate strength as one of the important requirements. A-7 steel, for instance, has a minimum yield point of 33,000 pounds per square inch. Steel manufacturers in developing steel to meet this requirement, from an economic standpoint, try to come as close to this minimum requirement as they can without having too great an amount of rejected material that does not meet the minimum requirement. In fact they use the probability factors to determine economically how close to this minimum they must shoot at in order to keep rejections to a minimum figure. Their general rule is to attempt to produce a steel that will exceed the minimum by one or two thousand pounds per square inch.

If mean values rather than minimum values are used it means that the engineer who designed the structure knows that a certain percentage of the material does not meet the minimum requirement, and in turn the minimum ultimate resistance of the structure can be lower than assumed in the design. It is difficult to conceive how the responsibility of a failure under such circumstances would not rest upon the shoulders of the design engineer. Mention is made that insurance underwriters use actuarial tables in determining the premium on life insurance policies. Whenever the number of deaths exceed these actuarial tables, the only involvement is the financial loss to the insurance company. A court of law would certainly not relieve an insurance company of the responsibility to meet its death claims just because of some circumstance which caused the death rate to exceed the actuarial tables based on probabilities; and it seems the same reasoning would prevail in placing the responsibility of failure of a structure in which the design was based on probabilities.

When a failure does occur, it usually develops that somewhere along the line the specifications were not followed, either in strength of material, type

of construction or the assumptions in the design did not include the load effect which actually occurred. Where does the engineer stand when he knew beforehand a certain percentage of the material did not meet the minimum requirements even though such a failure should not have occurred once in a century, based on the probability of failures?

With regard to the use of the term "maximum load effect" instead of "mean load effect" here again the design engineer is faced with the "facts of life." Assume the maximum load effect occurs on a structure that has been designed for the mean load effect; failure will probably occur. True this maximum load effect may not happen once in several hundred years, but that one time could happen the first year after the structure is put into service. Professor H. K. Stephenson has very ably presented the results of his studies of highway bridge live loads based on the law of chance.* The primary purpose of this study is not to develop static design loads but to evaluate the repetition of certain loads which can cause a fatigue failure. It cannot be left to chance that certain combinations of heavy loads will not occur in the lifetime of the structure when the human mind controls these sequences. The probability of occurrences based on statistics can be quickly upset when a fleet of trucks decides to use the convoy method of traveling the highways, or maintenance and construction work can pile up traffic to such an extent that a bridge can be greatly overloaded when compared to the probability of such a loading under moving traffic conditions.

Present day military loads are one of the prime factors in the design of highway bridges. Assuming the bridges would not be called upon to carry the military loads except in an emergency, the design load effect can be set up very closely to the maximum load effect using the minimum resistance to collapse. This example is cited as it is often said that loads approaching the live load factor used will seldom come on the structure.

It is not the intent to detract in any way from the value of the use of probabilities of survival and serviceability, as set forth in the report. It gives the engineer a tool with which he can better his guess and use engineering judgment to get the most for the dollar. It certainly has its place in the field of engineering design for determining the economic life of the structure where human lives are not endangered. It also can be the yardstick upon which the minimum factor of serviceability is based. In this case only the serviceability is affected and it is only an economic disaster when it is not satisfactory. It does not involve the collapse of the structure.

Perhaps the most important uses to be made of the study of probability theory in the design of structures are the factors that affect the probability of failures and occurrences. Certainly the study of the probability of occurrences adds greatly to the knowledge of the design engineer, but its misuse can place the responsibility of a catastrophe on the shoulders of the engineer.

OLIVER G. JULIAN,[1] M. ASCE and ALFRED M. FREUDENTHAL,[2] M. ASCE —The Committee is gratified by the discussion which the subject report has

*Proc. Paper 1314.

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evoked, although the number of discussors is smaller than might have been expected in view of the importance of the subject to all branches of the profession. The unfortunate fact that the large majority of engineers is not only unfamiliar with the basic concepts of the theory of probability and the statistical methods based thereon, but is reluctant to consider these subjects as relevant to their professional activity, probably provides the best explanation for the rather limited interest in the work of this Committee.

It must be admitted that the suspicion with which probability concepts and statistical reasoning are received by large groups of practicing engineers is not only the results of their lack of training in these disciplines, but frequently also of the insufficient training of some of those attempting to use statistical methods in engineering. Flagrant misuse of such methods leading to unjustified conclusions is, most unfortunately, quite widespread and has contributed to discredit an approach which, if correctly applied, would have been highly effective. The importance of probability concepts and statistics in most branches of engineering, and particularly in engineering design, can hardly be overestimated. Any understanding of the concept of structural safety dissociated from its inherent probability aspect must necessarily be incomplete. It is refreshing to note that the importance of this aspect is gradually being recognized in engineering curricula of our leading universities in some of which probability theory is now taught to all undergraduates.

Most of the discussors seem to be in favor of the Committee's approach to the problems of safety. Mr. Au points out that this approach is now even reflected in codes, as, for instance, in the ACI Building Code of 1956 in which the range of variation of concrete strength is considered in the specification of the design strength. Unfortunately even this new version of the Code fails to consider an integrated rational concept of safety. The introduction of load-factors only replaces the single conventional so-called factor of safety by two separate factors for dead (basic) load and for live load. It recognizes neither the impossibility of rationally specifying "load-factors" without reference to a specific set of loading conditions and their probability of occurrence, nor the irrationality of segregating safety with respect to material strength from that with respect to various load-effects. The specification of separate load-factors (based largely on guesses) instead of the evaluation of integral factors for unserviceability and for failure, based on probability considerations, in no way reflects the recommendations of the Committee and completely disregards the principal thesis of its report., i.e. the necessity of correlating factors of safety and factors of serviceability with probabilities of failure and probabilities of becoming unserviceable respectively. Without such correlation, the factors are devoid of definite meaning.

The basic discrepancy between the Committee's approach to structural safety and the specification of separate load factors is clearly and concisely pointed out in Mr. Walther's discussion, with the conclusions of which the Committee is in complete agreement. His statement that, in the light of probability considerations, separate load factors become in fact meaningless, and belong to a concept which is entirely different from that advocated by the Committee, is as pertinent as is his further remark that the lack of rationality of the load-factor concept is an inexhaustible source for argument and questionable contentions concerning the particular combination of load factors to be selected out of the infinite number of possible combinations pertaining to a specific probability of failure.

The Committee's principal thesis is further elaborated and illustrated in Mr. Basler's discussion which could well serve as an amplification of the Committee's report. Particularly the point concerning the difference in the minimum required factors of safety for single-load-path and multiple-load-path structures is well taken; it is a distinction well-known to aircraft designers and reflected in their rules of "fail-safe" design of airframes.

Mr. Walther has clearly recognized the basic difficulty in the probability approach, which is in the choice of pertinent probability distributions. The presentation, in the Committee report, of the discrepancies between factors of safety based on the normal and those based on the log-normal distribution is mainly intended to illustrate the irrelevance of the normal distribution in structural engineering problems. In the numerous series of observations of the variations of material properties as well as of loads reported by various authors in different countries, distributions closely approximating the normal were hardly ever encountered, while practically all observations can be fitted either by a log-normal or by one of the three types of asymptotic distributions of extreme values. Further regarding the normal distribution, one of its tails extends to minus infinity, whereas values below zero are impossible. The writer's own experience in attempting to fit theoretical distributions to observations of strength properties as well as to the values of loads suggests that the former are well represented by log-normal distributions provided they arise from production-processes subject to adequate control and inspection, but are better represented by the asymptotic distribution of smallest values in case the control is inadequate(1a) whereas the latter are best fitted by the asymptotic distribution of largest values.

Both Mr. Basler and Messrs. Vorlicek and Suchy favor the Gamma distribution (also known as Pearson type III) for the representation of the random variables significant in structural engineering; the latter illustrate the adaptability of this function by fitting it to observations of concrete compressive strength and of yield stress of reinforcing steel. While Mr. Basler's arguments in favor of this function are certainly valid, they apply equally well to the extremal or double-logarithmic distributions and to the logarithmic-normal distribution, both referred to in the Committee report. In fact it has been shown by E. J. Gumbel,(2) that within certain ranges of the coefficient of variation no distinction is possible between the extremal distribution on the one hand and either the gamma or the logarithmic-normal distribution on the other. In general, therefore, the choice between the three principal types of skew distribution has to be based on physical arguments rather than on statistical curve-fitting, particularly since the regions of principal interest are those of extremely low probability (tails of the distribution) which are outside of the range of direct observation. Different physical mechanisms will produce different distributions. "Extremal" phenomena, as for instance floods, fatigue and fracture will, by and large, produce extremal distributions; Cramer(3) has indicated a specific mechanism which produces a logarithmic-normal distribution, while one of the writers(1b) has illustrated a physical mechanism producing a gamma distribution. Of the three distributions the various types of the extremal distribution are undoubtedly the most versatile. Since tables for these distributions have been computed and published by the National Bureau of Standards,(4) their application is not difficult. The fact that these distributions are designed to represent extremes (smallest values of strength properties and largest values of load-effects) makes them particularly suitable for use in safety analysis which, basically, can only be concerned with the

occurrence of extremes and the estimate of the associated probabilities. The reason that the Progress Report does not also include numerical data pertaining to factors of safety based on extremal distributions is due to the fact that at the time of the completion of this report rigorous analytical methods for their evaluation were not available (the method by one of the writers,(1c) is only an approximation) rather than to any preference for the log-normal distribution. The analytical difficulties have, however, been overcome and values of factors of safety based on extremal distributions have recently been computed and will be included in a future report.

Mr. Walther's objection to the use of the mean value of the frequency curve of long-time traffic observations as the mean value of the design load is well taken. The Committee did not intend to recommend such use and endorses Mr. Walther's statement to the effect that the mean value of design load-effects should be taken as the mean value of probable maximum load-effects, the evaluation of which may well be based on statistical methods.

Mr. Ruble doubts that "the methods of attack covered in the paper will ever be accepted by specification writing bodies to establish definite factors for design purposes," presumably because of the "many practical factors which must be considered-" and because of the fact that the "---engineers responsible for the design and maintenance of structures ----- are going to base their recommendations on past experience." By discussing in some detail the various "factors" that have to be considered he provides, however, the most conclusive arguments for the statistical-probability approach to the concept of safety. It is fairly obvious that in the design of railroad bridges, with which Mr. Ruble is mainly concerned, the statistical variation of the static traffic loads is much less significant than in other types of structures, since the maximum train load will usually qualify as a load of fairly high frequency of occurrence, and considerations of future trends may outweigh all other considerations. In most other respects, however, Mr. Ruble's "factors" can hardly be dealt with rationally by any other but a statistical approach. How else can "past experience" be formulated and used for prediction of future variation? Statistics is, in fact, nothing else but a body of methods for making, on the basis of past experience, rational decisions in the face of present uncertainty.

Thus, for instance, the dynamic load-effects which are subject to the many uncertainties enumerated by Mr. Ruble can certainly not be dealth with by functional analysis. Hence no other alternative exists but to collect the "past experience" embodied in the field test results and to present it statistically in such a manner that it can be used for prediction of the expected range of variation, as a basis for "rational decisions" in connection with the design of structures for future service. (The clause underlined above cannot be emphasized too strongly.) Such an approach was recommended some thirty years ago by Professor N. Streletzky,(5) one of the foremost Russian railroad engineers of his time.

After listing the many uncertainties characteristic of the material properties, all of which are currently dealt with by statistical methods following recommendations by the ASTM and other organizations, Mr. Ruble turns around and asks "what will be gained by making a statistical study of the variation of the static loads crossing the structure?", as if this were the only subject the Committee report was concerned with. With respect to railroad bridges the answer to Mr. Ruble's question is clearly: "nothing will be gained since these variations are immaterial in this case." But as pointed

out by one of the writers,(1d) railroad bridges represent that special type of structure the static design load of which is essentially identical with the maximum static load pattern. This fact, however, does not invalidate the probability approach to the safety analysis of such bridges, as there are many other uncertainties besides that of the static weight of the traffic load, most of which have been listed by Mr. Ruble.

In his closing remark Mr. Ruble states that "the railroad committees are in no position at this time to say what factor of safety should be used for railroad bridges." Admittedly, it is not an easy task to establish such a factor; if it were, neither the ASCE nor the British and other European Engineering Societies would have found it desirable to establish committees to attempt to deal with this problem on a rational basis. It is, however, a safe prediction that the railroad committees will remain in the same position unless they avail themselves of the modern rational methods. It appears advisable to select probabilities of failure which are considered sufficiently low, for example 10^{-6} or 10^{-7}, and then compute factors of safety from these probabilities and the other pertinent parameters.

There is only one point in which Mr. Archibald appears to dissent from the approach to safety expressed in the Committee report, and that is in his belief that "where danger to human lives is involved in the design of structures" the definition of the minimum required factor of safety "should be the ratio of the minimum estimated resistance ---- of a large number of (nominally identical) structures and the maximum load-effect for which the (particular) structure is designed." A prime objection to this formulation of the factor of safety is the subsequent definition of the maximum design load as "the maximum load that a structure would sustain at or before failure," in other words as the carrying capacity of the structure, which is "usually arrived at through the use of the live load factor --- equivalent to two, three or four times a design load using working stresses." By this definition of the maximum design load the factor of safety appears to be made a function of those working stresses the use of which it is supposed to replace, or of an arbitrarily selected load factor the concept of which is incompatible with the probability approach to structural safety advocated by the Committee. The implication would appear to be that the use of a load factor "equivalent to two, three or four times a design load" (the relation of which to the actual loading conditions is left unspecified) provides a more reliable approach to the estimation of a maximum load than the use of statistics based on past observations. Only a strong belief in the inapplicability of the statistical approach to load-analysis could justify such a conclusion. Mr. Archibald's discussion of this aspect of the problem is, in fact an expression of such belief, which is backed partly by the argument that "it cannot be left to chance that certain combinations of heavy loads will not occur ---- when the human mind controls these sequences," and partly by a suspicion, based apparently on experience with misapplied statistical arguments, that statistical theory is unable to describe or predict "the facts of life."

There can be no quarrel with the argument that "when a fleet of trucks decides to use the convey method" considerations of probabilities of occurrence of load combinations are useless. If such methods become accepted, the design assumptions of highway bridges will necessarily resemble those of railroad bridges, with a "standard train" made up of unbroken sequences of the heaviest trucks on all lanes as design load.

The suspicion that statistical theory, wherever basically applicable, contradicts "the facts of life" is, however, unfounded. Only when the selected method is in error do the conclusions based on it contradict reality. Thus, the apparent discrepancy between the low "a priori" probability of occurrence of a certain heavy load unit and the surprisingly high incidence of relatively long sequences of such units can be resolved by the application of the relevant statistical theory of "recurrent events" or "runs" to all occurrences during the estimated service life of the structure or parts thereof, rather than that of the binomial or Poisson distribution to individual occurrences, to the indiscriminate use of which Mr. Archibald rightly objects. For instance, with an assumed average frequency of 5 heavy trucks per 100 vehicles and a traffic volume of 500 vehicles per hour, the probability of at least one "run" of 4 consecutive trucks in 24 hours is as high as 0.069, which means that "runs" of at least 4 trucks must be expected to occur at an average once every two weeks; with an assumed average frequency of 10 heavy trucks per 100 vehicles an average of two such runs is predicted in the course of every three days, while runs of three consecutive heavy trucks are to be expected once every five hours, which seems a surprisingly high incidence. These so-called "return periods" of runs of at least 4 heavy trucks are obviously much too short in relation to the expected life of a structure carrying such traffic to qualify as maximum design loads (unless the structure cannot accommodate more than four trucks) although on a purely statistical basis the "return period" of structurally significant runs of at least 4 trucks, that are runs at closest spacing, is much longer than that estimated above and valid for average spacing. However, with respect to the use of statistics to the prediction of traffic pile-ups, Mr. Archibald's objections are probably valid, since such pile-ups caused by maintenance and construction are likely to be much more frequent than predicted by the statistical theory of "random bunching."

Without statistical consideration of the pile-ups in terms of vehicle distance, the relatively high frequency of occurrence of long sequences of heavy units predicted by the statistical theory of runs shows that this theory not only represents the "facts of life" quite adequately but leads to conclusions which, on the basis of actual short series of observations, would be considered as rather "unlikely," and thus to an apparent over-estimate rather than an underestimate of the probabilities of extreme loading sequences of structural significance. In fact, the surprisingly frequent appearance of relatively long sequences of unlikely loads might give rise to the unwarranted conclusion that such sequences are the effect of deliberate control, while they are nothing else but the result of perfect randomization. This phenomenon, with respect to other random events, has been well-known to statisticians since the second decade of this century, when the statisticians v. Mises in Berlin and v. Bortkiewicz in Vienna combated various attempts by philosophers to introduce special "laws" of "serial occurrences" or bunching," to explain the surprisingly high observed frequencies of occurrence of long sequence of events the a priori probabilities of which seemed to exclude the possibility of such occurrences on a purely statistical basis.(6)

Quite independently of whether or not the safety of the structure involves danger to human lives, unless load sequences are arbitrarily controlled, as in the case of railroad trains or truck-convoys, no valid objections can be raised against the use of statistics based on past observations for the prediction of future load-effects. The maximum load-effect is therefore usually associated with a probability of occurrence which, when combined with the

probability of simultaneous occurrence of a carrying capacity lower than this maximum load-effect, will determine the probability of failure of the structure, under the assumption of an ultimate design with a factor of safety of one. In principle this approach does not differ from the approach outlined in the Committee's report, based on mean values. In either case, one must realize that if a large number of cases are considered a certain percentage of the material in place will not meet the design requirements, while a certain probability of occurrence of load-effects higher than that used in design will always exist, unless the structure is loaded to capacity with the heaviest possible loads, a condition which appears rather unrealistic for any, but relatively short-span structures, and is certainly not considered in the current design specifications of medium or long span structures. Whether this percentage or probability is high, as in the case of mean design values, or very low, as in the case of extremal design values, affects only the numerical value of the resulting factor of safety (which is high in the case of mean design values and low in the case of extremal values) not the inherent safety of the design measured by its probability of failure, since in either case the probability of failure can be made the same. The assumption that absolute maxima and minima for load-effects and for strength, respectively, can be selected so that the probability of failure becomes not only extremely small but actually zero, would lead to designs incomparably more conservative than those resulting from current specifications. The assumption that designs can be based on absolute maxima or minima, with an associated zero probability of failure is simply an illusion. In this connection reference may be made to the second paragraph of the Committee report.

The fact that actual failure rates of structures are extremely low, only suggests that the probability of failure implicit in current design practices, whether quantitatively evaluated or conveniently disregarded, is sufficiently low. Ordinarily, it is not the designers responsibility to assess the probable safety of his design, beyond the necessity of meeting the specification requirements; this assessment should remain the responsibility of the specification writing bodies. It is however, the duty of those bodies to establish a rational measure of safety with the aid of which the safety of various designs or the comparative safety of the various parts of one and the same design can be compared. Such a measure is provided by the probability of failure. Rather than pretend that in good designs this measure can and should actually be eliminated (because of the implication that a structure designed on its basis is not absolutely safe) in rational design specifications this measure should be utilized to achieve a reasonable balance of safety throughout. Such a balance cannot be achieved by the simple guess work procedures upon which current safety or load-factor specifications are based. In choosing values for probability of failure, or of becoming unserviceable, the type of failure and its consequences should, of course, be given consideration.

The Committee expresses its thanks to all discussors for their important contributions.

REFERENCES

1. Safety and the Probability of Structural Failure, by Alfred M. Freudenthal, M. ASCE, Transactions Vol. 121, 1956.

   a) pages 1358 and 1359

b) page 1350

c) pages 1370 and 1371

d) page 1343

2. Statistics of Extremes by E. J. Gumbel, Columbia University Press, New York, 1958, page 179 ff.

3. Mathematical Methods of Statistics by Harald Cramer, Princeton University Press, Princeton, N. J., 1946, page 220.

4. Probability Tables for the Analysis of Extreme-Value Data, U. S. Department of Commerce National Bureau of Standards Applied Mathematics Series No. 22, 1953. Statistical Theory of Extreme Values and Some Practical Applications, U. S. Department of Commerce, National Bureau of Standards Applied Mathematics Series No. 33, 1954.

5. Impact on Railroad Bridges, by Prof. N. Streletzky, Proc. First Int. Congress of Bridge and Structural Eng., Vienna 1930.

6. Probability, Statistics and Truth by Richard Von Mises, Second Revision English Edition, the Macmillan Company, New York, 1957.

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LATERAL BRACING OF COLUMNS AND BEAMS[a]

Closure by George Winter

GEORGE WINTER,[1] M. ASCE.—The lively response to this paper is greatly appreciated by the writer as an indication that his contribution will be of use to the profession. The writer is gratified not only by the questions asked by some of the discussers but particularly by the fact that two of the discussions considerably extend the range of the methods he has proposed.

Mr. G. G. Green's contribution supplements the information developed by the writer in that it permits one to compute the load capacity of a column if the bracing system supplies less than "full bracing." In this case use must be made of the detailed theory of elastically supported columns, for instance in the form presented in Fig. 9 of the writer's paper. (That figure, as stated in the paper, was taken from Refs. 3 and 6 of which Mr. Green was a co-author while doing graduate work at Cornell University, a fact which he modestly failed to mention in his contribution.) The reader may be somewhat confused by two evident misprints in the first paragraph of Mr. Green's contribution: His Fig. 1 is a re-plot in terms of P_e of the writer's Fig. 9 (rather than of his Fig. 1, as mistakenly stated). Mr. Green has chosen, however, only to replot the last of the various sloping portions of the respective curves given on the writer's Fig. 9. In order to use Mr. Green's method for all possible values of $k_{act} < k_{req}$, it is necessary to have the complete information on elastically supported column for the entire range from $k_{act} = 0$ to $k_{act} = k_{req}$. This information is contained in the writer's Fig. 9 and is given in greater detail and with sufficient accuracy for design purposes in Ref. 3.

Mr. Green's method for "less than full bracing," as illustrated in the second part of his numerical example, can be stated in general terms as follows:

Given the column, point-supported at n points, and the value k of these elastic supports.

1) Compute the actual strength S_{act} of each of the braces, i.e. the maximum reaction from the column which it can withstand.

2) From this, compute the maximum deflection $d_{max} = S_{max}/k$ to which the brace can be subjected without failure.

3) For the given k and the given column dimensions, find the critical load P_{cr} of the "ideal" elastically supported column. This can be done approximately from the writer's Fig. 9, and with better accuracy from Ref. 3 (i.e.

a. Proc. Paper 1561, March, 1958, by George Winter.

1. Prof. and Head, Dept. of Structural Eng., Cornell Univ., Ithaca, N. Y.

from the equations on pp. 11 to 17 or from Figs. 11 and 26 of that reference).

4) The maximum load on the column P_{max}, is now determined from the requirement that the bracing deflection caused by this load shall not exceed d_{max} as determined in step (2). For this purpose the first equation in Mr. Green's contribution is utilized which, in terms of the present notation, is

$$d_{max} = d_o \frac{P_{max}}{P_{cr} - P_{max}}$$

From this it follows that

$$P_{max} = P_{cr} \frac{d_o}{d_o + d_{max}}$$

The allowable load is then obtained by dividing P_{max} by the safety factor, as indicated in Mr. Green's example.

(It should be noted that the equation $d = d_oP/(P_{cr}-P)$, utilized by Mr. Green as indicated in step (4) above, to the writer's knowledge has not been rigorously proved for elastically supported columns. However, apart from being a close approximation in most elastic stability phenomena, its applicability to elastically supported columns has been demonstrated by some 28 tests reported in Ref. 3, covering a wide range of variables.)

It is interesting to note the following from Mr. Green's numerical example: The channel column, when "fully braced" according to the writer's requirements, would have an allowable load of 55,500 lbs., as shown in the first part of his example. Mr. Green shows that if the bracing which is actually furnished has a stiffness as small as one-third of that required for "full bracing," the allowable load is only reduced to 34,500 lb., or to 62% of the fully braced value. It is seen, however, that deflections in this "partially braced column" are very considerably larger than when "fully braced." Thus, while the fully braced column in Mr. Green's example, under its capacity load P_{ult} = 105,500 lbs. has a deflection of only 0.36" or 1/1000 of its length, the "partially braced column" under its much smaller capacity load of 65,500 lbs. has a deflection of 1.07 in., three times the previous value, or 1/340 of its length. For this reason, when using Mr. Green's method for "partially braced columns," the following step should be added:

5) Check whether the section of the column is capable of simultaneously supporting without failure the axial load P_{max} and the bending moment $M_{max} = P_{max}d_{max}$.

It is believed that Mr. Green's example illustrates the advantage of the writer's proposal to design for "full bracing." This is seen to result not only in a stronger but also in a much more rigid column. It is for this condition that the writer has developed his very simple method. Mr. Green's complimentary approach permits one to ascertain the effect of "partial bracing" with relative simplicity, if such partial bracing is encountered in some special situation, such as when checking an existing structure.

Mr. M. A. Larson extends the scope of the writer's investigation in another direction. He points out that in many cases bracing to compression members or flanges is provided in a manner different from that analyzed in the paper. There, support was furnished by the equivalent of elastic springs acting independently from each other. Mr. Larson points out that if bracing is provided by rigid diaphragms (floor or roofs) spanning parallel to the to-be-braced

members and supported at the ends by shear walls or other rigid parts of the structure, then a different kind of action ensues. Such a system is schematically represented by the writer's Fig. 8, or Mr. Larson's analogous Fig. 1, (the jacks at B and C being removed). The 18 WF 50 girder AD carries the floor beams BE and CF. These, in turn, support the cellular steel decking which is interconnected to form a rigid diaphragm and is attached at its ends to the shear walls which pass through A and D. It is evident that the top flange of the girder can buckle only if the entire diaphragm distorts as shown by Mr. Larson. This case indeed appears to be somewhat different from the writer's "spring supports" (see his Figs. 2 through 7), but can be readily reduced to it as will be shown below. Mr. Larson simplifies his system for purposes of analysis by assuming that the diaphragm's entire resistance to distortion is furnished by its shear rigidity. In numerous diaphragm tests at Cornell University that share of the total deflections which was caused by shear distortion varied from about 2/3 to 5/6 of the total deflections, the rest being due to flexure of the deck plus marginal members acting as a plate girder. The assumption of assigning the entire deformation to shear is, therefore, reasonably justified.

Mr. Larson implies that by far the most frequent type of actual bracing is of this shear diaphragm variety, and that the writer's "elastic braces connected at the far ends to points fixed in space" hardly occur in actual structures. The writer is well aware that on the West Coast, in view of seismic requirements, rigid shear diaphragms are almost universally used for floors and roofs. Under these conditions it is evidently most economical to use these same diaphragms also for bracing against buckling. However, in non-seismic construction roof elements (such as standard steel roof decks, gypsum or concrete plank, etc.) are not usually connected to form rigid diaphragms. Bracing in mill-type buildings is then furnished by providing braced bays every so often and by bracing compression members in the other bays, such as top chords or top flanges of roof trusses or girders, by connecting them to these braced bays along specified lines (such as by purlin struts). The same kind of bracing is furnished for the columns in regard to buckling in the direction of the non-rigidly sheathed walls, by connecting girts or similar longitudinal bracing members to the braced bays. Since the rigidity of the braced bay is very large as compared to that of the to-be-braced members, the writer's independent spring supports are closely approximated and his method permits the dimensioning of the bracing members just mentioned. Numerous other such situations occur, such as the inner leg of a mill building column mentioned in the writer's introduction.

The conclusion to be drawn is that both Mr. Larson's and the writer's bracing systems occur, depending upon the specific situation.

Turning now to Mr. Larson's apparently different treatment of the writer's example, it will be shown (a) that his treatment reduces identically to the writer's, (b) that it is by no means irrelevant, contrary to Mr. Larson's contention, whether buckling is assumed to occur in the mode of his Fig. 3a or that of Fig. 3b. While the two result in the same required shear rigidity as defined by Mr. Larson, only the mode of Fig. 3b results in the maximum, and therefore governing, strength requirements for the bracing system.

Referring to the writer's Fig. 5b it is seen that the shear distortion in the two exterior spans is d/L and in the interior span 2d/L. Consequently, using Mr. Larson's shear rigidity γ', the shear forces in the corresponding portions of the diaphragm are

$$V_{ext} = \gamma(d/L) \qquad V_{int} = 2\gamma(d/L) \tag{I}$$

From this it is seen that the reactions between girder and floor beam are

$$F = 3\gamma(d/L) \tag{II}$$

in terms of Mr. Larson's shear rigidity γ, while the same force is

$$F = k\,d \tag{III}$$

in terms of the writer's spring constant k. Hence, if bracing is provided by the shear rigidity of the diaphragm rather than by exterior, elastic bracing members, then for the buckling mode of Fig. 5b, equating the two above expressions for F, one finds

$$\gamma = k(L/3) \tag{IV}$$

which, for the case at hand, gives the relation between Mr. Larson's γ and the writer's k. For the buckling mode of Fig. 5a, similar simple reasoning leads to

$$\gamma = kL \tag{V}$$

For the latter case it was found that $k_{id} = P_e/L$ and for the former $k_{id} = 3P/L$ (see writer's Eqs. (a) and (6)). If these values are substituted, respectively, in Eqs. IV and V, above, one obtains for both cases

$$\gamma_{id} = P_e \tag{VI}$$

which is identical with Mr. Larson's Eq. (1). In fact, Mr. Larson's general statement that the shear rigidity γ_{id} is independent of both the number of spans and the buckling mode appears to be correct, as is easily checked in the above manner for other cases.

However, his statement that the buckling mode is also irrelevant in regard to the required strength of the bracing system is erroneous, as is easily demonstrated by the same example. It is first necessary to specify what is meant by "strength" in the case at hand. For the given system to function adequately, (a) the shear strength of the diaphragm proper and (b) the strength of the connections between floor beams and girder must be adequate, individually and separately. Neither of these is identical with the quantity S_{req} computed, but not defined, by Mr. Larson. For the present purposes call V_{req} the shear strength of the diaphragm and as before in the writer's paper, S_{req} the strength of the beam-to-girder connection.

Then, for the case of the initially distorted top flange one obtains, for both Mr. Larson's Figs. 3a and 3b

$$\gamma_{req} = 382 \text{ kips/radian}$$

using either Mr. Larson's method or the writer's values and Eqs. IV and V, above. The maximum shear force on the diaphragm is now obtained by substituting this value in the expression for V_{int} (see Eq. (I), above), which gives

$$V_{req} = 4.58 \text{ kips.}$$

The maximum required connection strength is obtained by substituting γ_{req} in Eq. (III) for the interior reaction F, which gives

$$S_{req} = 6.86 \text{ kips}$$

This is identical with the writer's original value for the case of $k_{act} = k_{req}$ (or in Mr. Larson's terms, $\gamma_{act} = \gamma_{req}$). Neither of these values checks with Mr. Larson's S_{req} = 2.3 kips, which is seen to be unsafe.

If instead of the mode of Fig. 3b that of Fig. 3a is investigated, similar elementary computation shows that one-half the above value would have been obtained for V_{req} and one-third for S_{req}, values which are also evidently unsafe. It is thus seen that the investigation of the correct buckling mode is just as necessary for bracing by shear diaphragms as for bracing by elastic springs, contrary to Mr. Larson's contention.

It is not maintained that the above disposes of the general problem of shear diaphragm bracing. In particular, Mr. Larson's Eq. 1 (Eq. VI, above) is intriguing in its simplicity. Also, cases of three and more interior supports may not as easily reduce to the corresponding cases of spring supports. Finally, it is admitted that the way in which the writer used the quoted results of the diaphragm test to derive k_{act} = 100,000 lb./in. for his numerical example may have been excessively conservative for the given buckling mode.

In regard to continuous support, Mr. Larson first proposes a different method for finding P_{cr} in conformity with classical theory. Whether one prefers Mr. Larson's or the writer's method, or whether one prefers to use a curve such as on the writer's Fig. 9, is a matter of taste, since all three methods are simple and give identical answers within engineering accuracy.

In contrast to the point-supported case, Mr. Larson's analysis of a compression member continuously supported in a medium which possesses only shear rigidity is very interesting indeed and leads to results of great simplicity and relevance. While the writer has not checked all the ramifications of this problem, he believes the method to be correct and thinks that this portion represents the main contribution of Mr. Larson's discussion.

In passing the writer wishes to draw attention to the lack of published data on strength and rigidity of diaphragms, particularly those made up of cellular steel panels and of steel roof decks. A tremendous amount of full-scale testing of such diaphragms has been done in recent years, mostly on the West Coast, in part at Cornell University, and in other places. The fact that all these tests were made on proprietary products, so far has prevented making the results available to the profession at large. The present investigation (both Mr. Larson's contribution and the original paper) illustrate to what extent such data are needed for intelligent and economical design. Some way ought to be found to make these data available to the profession in a form which would be useful for design without violating justified, competitive proprietary interests.

Prof. B. G. Johnston's complimentary remarks are much appreciated. He invites the writer's additional comments on a specific question, namely, to what extent, if any, mere friction between a floor and the supporting beam should be relied upon to provide lateral support to the latter and, at the other extreme, what justification there is, if any, in the frequent textbook statement that flange embedment in concrete is the minimum requirement for acceptable lateral support.

The writer's numerical example of an 18 WF 50 girder of 30 ft. span shows indeed, as stated in the paper, that "mere friction would suffice to develop the force required for lateral bracing" for the loads and dimensions of this specific example. The very next sentence, however, reads: "This is not to suggest that such friction should actually be relied upon . . ." and goes on to indicate one of several minimum shear key measures which will provide more than adequate support even for zero friction. It is clear that the proposed method of calculating lateral support requirements must be used in conjunction with, rather than as a substitute for engineering judgement. This means that the action of each bracing system must be correctly reflected. For instance, the timber planking mentioned by Prof. Johnston will ordinarily not be interconnected to form a rigid diaphragm and, therefore, may not have any bracing value of its own, in contrast to a monolithic concrete floor. On the other hand it is believed that the proposed method is sufficiently reliable to answer Prof. Johnston's questions as follows:

The analysis as presented definitely shows that adequate lateral support is easily obtained by measures much less restrictive than the embedment of the flange in concrete. That textbook statement was an acceptable and conservative advice as long as no positive, quantitative information was available, but should be discarded as excessively restrictive. On the other hand, as indicated by the two quotations above, the writer would not rely on friction alone even if computation indicates this to be adequate. He would provide a minimum of shear connections in conformity with calculations, by whatever means appear most economical in the given case, such as welded studs, light bolts, special clips, nailing where such is possible (as in many light-gage steel shapes) and the like.

Prof. William Zuk lists a number of special situations for which he would wish more accurate methods used than those proposed by the writer. He points out that theoretical analyses of some of them are given in his own paper (which was duly quoted as Ref. 4 by the writer), but generally in a form too complex or too special for design use. He concludes by saying that "The contribution is a valuable one but perhaps not a final one." The writer would like to add that no contribution is ever final and that it is precisely this fact which makes for the zest in research as an occupation.

It is repeatedly stated in the paper that the writer's sole purpose was to develop a reliable approximate method sufficiently simple for the usual design purposes. To take an analogy, there are many situations in which the presently accepted parameter Ld/bt for lateral buckling is inapplicable or misleading. Yet it is valuable if used with discretion, and is a better crutch than we have had before. It is hoped that the present paper, likewise, will be found to be a better crutch than we have had before. The writer fully agrees with Prof. Zuk that this should in no way stop further and more refined research into the various phases of the bracing problem. He also agrees that his methods were never intended to apply to the problem of compression chords or flanges of open through bridges (pony truss bridges and the like), except possibly as approximations in the preliminary design. A considerable, and very adequate, body of literature exists on this special, important problem.

Mr. A. Chibaro's complimentary remarks are much appreciated.

Mr. F. E. Fahy, Technical Adviser, Bethlehem Steel Company, Bethlehem, Pennsylvania, in an inquiry received too late for publication, asks the follow-following:

"In dealing with cases of multiple support, such as in Figs. 5 to 7, or of continuous support, should not the value of the initial amplitude d_O, to be used in computing the required strength of the restraining means, be that for the half wave of the buckled shape or column? This is as illustrated in Figs. 6 and 7. The problem arises in dealing with a long strut that is initially bowed in a single half wave of considerable amplitude. A slender strut with an overall length of 20 ft. might conceivably have an initial deflection at its mid-point (d_O in Fig. 4) of something like 3/4 in. However, if it is supported at several points, as in Figs. 6 and 7, should not the value of d_O to use in Eqs. 17 and 25 be some appropriate fraction of the total 3/4 in. amplitude?"

This question is really outside the writer's competence, since its answer depends more on rolling, fabrication and erection practices than on analytical considerations. For multiple support the most unfavorable and analytically most convenient assumption has been made that the initial distortion is affine to the buckling shape (see e.g. Fig. 6). It was stated that this is an assumption of low probability and, therefore, quite conservative. If it were true that long struts are likely to be bowed initially only in a single half wave, as implied by Mr. Fahy, it would evidently be justified to take an appropriate fraction of the total amplitude for d_O in case of multiple support. On the other hand, straightness tolerances usually are so worded that they do not rule out S-shaped members with maximum amplitudes at one or both crests equal to the full tolerance. In testing as-received, rolled members the writer has repeatedly found them to be S-shaped, although their amount of crookedness has rarely approached the value of the full straightness tolerance. Also, in fabrication and erection, additional crookednesses of single or multiple wave shape are likely to be superposed on those of the member as-rolled.

This emphasizes the fact that, as design procedures are being refined, greater attention is needed to tolerances of shape, and that this is a matter for those responsible for design specifications and codes, as repeatedly indicated in the paper. Pending such determinations, those connected with rolling, fabricating and erecting, as Mr. Fahy himself, are better able to judge what conditions are likely to obtain in the finished structure than the writer.

It is evident from the analysis presented in the paper that for multiple support the required strength of the bracing is considerably less if the initial shape is a half-wave of amplitude d_O than if it is a multiple wave curve affine to the buckling mode, with the same amplitude d_O. If the writer were to use his method for design, without appropriate guidance by codes, he would, therefore, use for the determination of d_O the total length of the member divided by the number of intermediate supports (i.e. 3L/2 for Fig. 6, 4L/3 for Fig. 7, etc.). Taking this length as basis, he would then determine d_O as two to three times the straightness tolerance stipulated for that length in the pertinent design code or specification. To take such a multiple appears necessary to account for the possibility of actually S-shaped members, for the additional influence of possible eccentricities, and for such other imperfections of alignment as may be caused in fabrication and erection.

ACKNOWLEDGMENT

The substance of this paper was first presented orally in a course of lectures which the writer gave at the University of Liege, Belgium in the spring

1957 during a sabbatical leave in Europe, made possible in part by the award of a John Simon Guggenheim Memorial Fellowship.

Typographical Error: Eq. 26 of the original paper should read

$$P_e = P_E(n + 1)^2$$

MOMENTS IN RESTRAINED OR CONTINUOUS BEAMS BY THE METHOD OF PARTIAL MOMENTS[a]

Closure by Harry Posner

HARRY POSNER,[1] M. ASCE.—The author is very thankful to the engineers whose discussions appeared in the Journal of the Structural Division in the following order: Messrs. William Walker and Joseph T. Kolibal in the July issue, Mr. A. A. Eremin in the September issue and Mr. Lembit Kald in the November issue. It is a source of gratification to the author that busy engineers have taken the time to test a new method of approach to the analysis of continuous beams in order to appraise its value in practical application.

It is noted that Messrs. Walker and Kolibal feel that the application of the direct "Moment Distribution Method" which they had been using for a number of years gives results more quickly than the proposed method. It is regrettable, however, that they have not indicated the manipulations they had to go through in order to set up their constants. It is also regrettable that they had not carried their computations to a point where the moments are expressed in terms of the loading "W" but have instead applied their constants to arbitrarily assumed values of M_{C-D} and M_{D-C}. It is interesting to note that Mr. Lembit Kald, using the direct "Moment Distribution" method found the "Partial Moment" method worth while as a time saver. Of course, it is understandable that procedures to which one has become accustomed through repeated usage would seem and be more expeditious than a new method applied for the first time.

Mr. A. A. Eremin's endorsement of the "Partial Moment" method is very gratifying, indeed. The graphical construction shown in Fig. 2 of Mr. Eremin's discussion can no doubt be used to good advantage for the purpose of obtaining the "Partial Moments" (Step Two in the author's solution). The engineer is now given three choices for getting the "Partial Moments," namely:

1. The analytical method offered by the author.
2. The graphical method offered by the author.
3. The graphical construction offered by Mr. Eremin.

In the final analysis it will be up to the engineer at work to make his choice of tools when faced with the task of solving this type of problem.

The author feels particularly indebted to Mr. Lembit Kald for taking pains to set up the solution of a problem in two parallel columns, one for the author's method of "Partial Moments" and the other for the "Direct Moment Distribution" method now in general use in one form or another. That he found the author's method (using the analytical solution) attractive and time saving is very encouraging.

a. Proc. Paper 1567, March, 1958, by Harry Posner.
1. Cons. Eng., formerly Engr. of Structures, N.Y.C.R.R. System.

It is understandable that a certain amount of resistance must be overcome in launching any new and unfamiliar method or procedure in the field of applied mechanics as in other fields. The old admonition of Alexander Pope:

> "Never be the first by whom the new is tried,
> Nor yet the last to lay the old aside."

Still holds true and perhaps always will.

It is hoped that the proposed method will contribute something of value to the busy engineer. In his own practice the author has used the method of "Partial Moments" for a good many years and found it to be extremely useful. This is particularly true when dealing with problems involving trial loading investigations. The greater the number of spans covered by the continuous beam the more time saving the method becomes. The ease with which the answer can be expressed in terms of the loadings "W" or "P" adds to the usefulness of the proposed method.

NUMERICAL SOLUTIONS FOR BEAMS ON ELASTIC FOUNDATIONS[a]

Closure by Henry Malter

HENRY MALTER,[1] M. ASCE.—The author wishes to thank those who participated in the discussion. The interest in the subject, as made evident by the response to the particular article, as well as to the many other recent articles on the same subject, indicates a growing realization on the part of many that a solution to the elastic foundation problem may be readily obtained, and should not be avoided.

The author was particularly interested in the many additional techniques that the discussion brought forth, although it becomes quickly evident that many of the methods presented are but variations of the four general methods a) solution of the differential equation, b) successive approximations, c) finite difference method, d) Fourier Series analysis.

Mr. Dodge's approach stems from the general solution of the basic differential equations, and the curves which he has presented are undoubtedly of value to one who is familiar with their derivation, and may be used to supplement the similar type charts appearing in "Advanced Mechanics of Materials" by Seely-Smith. It was of interest to read that Mr. Dodge applied the solution to a floating bridge type of structure. The author recalls the application of "elastic foundation" methods to a floating ore handling structure. It is true that on a water support a "Winkler" type of foundation is present, and any of the above methods may be used to advantage.

The discussion by Mr. Ryker touches on the subject of the difficulty of convergence of the Vianello-Stodola type of solution. This subject of convergence can become rather sticky, and was not discussed by the author, although one of the techniques for more rapid closure was employed, as correctly pointed out by Mr. Ryker. However, it is felt that the method of finite differences is so much easier of application, that the entire convergence problem can be bypassed - unless one is interested in the problem as a mathematical digression.

The discussion by Mr. Reese is again a variation of the finite difference method, and one which Mr. Reese emphasizes may be easily applied through the use of a digital computer. Although it is recognized that the American engineering profession is at present very digital computer conscious, it should be remembered that there are many engineers both in the United States and particularly in foreign countries, who may never have access to this latest engineering device. It should be emphasized that the entire calculations for the author's original paper were all performed with nothing more than a ten inch slide rule - not even a desk calculator was available at the time.

a. Proc. Paper 1562, March, 1958, by Henry Malter.

1. Head, Dept. of Civ. Eng., Robert College, Istanbul, Turkey.

The reference mentioned by Mr. Bergman was deliberately omitted from the author's original article in an attempt to play down the application of these techniques to building foundations - where they rarely belong. It is unfortunate that the field is entitled "Beams on Elastic Foundations." A substitution of the word "springs" for the word "foundations," would separate the field from building foundations, and relegate it to where it properly belongs - the "Winkler" type of support. An excellent approach to this phase of the subject may be obtained in a discussion by Mr. W. E. Hanson[2] to the original article by Professor Popov.

2. Discussion article by W. E. Hanson, Proceedings Volume 77, April, 1951.

SPECIFICATIONS FOR PRESERVATIVE TREATMENT OF TIMBER: PROGRESS REPORT OF A SUBCOMMITTEE OF THE COMMITTEE ON TIMBER STRUCTURES[a]

Closure by the Committee

The appreciation of the committee is herewith expressed to Messrs. O'Brien and Reno for their interest and efforts to improve upon the information offered in the original paper.

Mr. O'Brien comments that a 14# retention is practically the optimum amount of creosote solution that can be injected into Douglas Fir, whereas 20# is the approximate upper limit for Southern Pine. He correctly states that the difference is primarily due to the treatable characteristics between the species. The intent of this specification was not to discriminate between species but to suggest that both species be treated to refusal when used in coastal water locations. The 14# retention for Douglas Fir has been found adequate for such severe exposures by actual service records.

There was no intent to imply that difficulty is encountered in treating Southern Pine or any other species. The intent of Table I was only to suggest minimum net retentions adequate to protect against the various destructive agents.

Mr. Reno objects rather strenuously to the statements made in the report to the effect that the only effective treatment against wood-destroying organisms is some type of pressure treatment.

Mr. Reno may not be aware of the fact that this report emanated from Sub-Committee 7 of the "Committee on Timber Construction," and as such, the committee members had an immediate unanimity of opinion that their subject matter encompassed only "engineered timber structures." This opinion thus excluded (perhaps incorrectly) discussion of dip and swab treatments, paints, etc. that are commonly used in dwellings, millwork, sash and door and similar mild exposure conditions of use. The Report admittedly does not specifically describe this limitation and perhaps should incorporate a more descriptive title prior to final inclusion in Manual 17.

Mr. Reno's statement regarding "high resistance to decay is inherent in the heartwood of California Redwood, Tide water Red Cypress, and various cedars," is true, but relative only. The phrases, "durable species," "decay resistant species," "moderate decay hazards," are overworked and also only relative, and are not now considered definitive enough to incorporate in a specification.

Mr. Reno's statement regarding the lack of decay in wood having a moisture content not exceeding 20% is also true. Here again the Report is not solely concerned with destruction of timber by decay fungi, as Mr. Reno

a. Proc. Paper 1637, May, 1958.

evidently assumes. If these same timbers that he would not treat by any method simply because their moisture content remained below 20% are exposed to termite or other insect attack, it is believed that Mr. Reno would agree that treatment of some sort would be advisable.

In summary, the Committee would like to note some corrections and additions to bring the Report up to date. The technological advances in wood preservation within the period of preparation and publication of the Report have almost rendered parts of the recommendations obsolete. However, the only major change to be noted here would be the addition to Table I, under "inorganic Salt" the preservative:

Chromated Copper Arsenate (Erdalith or Greensalt)

For use under moderate loading conditions: -0.35 #/ft.^{3}

For use in contact with ground water: -0.75 #/ft.^{3}

Other corrections to be noted are:

Page 1637-2
Under "Preparation for Treatment"

Line 1 ". . . pressure tested . . . ." should be . . . pressure treated. . . ."

Page 1637-3
Under "Incising" Sentence 1

"To insure uniform penetration, all sawn material of refactory species, 3'' or thicker should be incised on all faces."

Page 1637-4
Under (4) Fire Retardant Chemicals: Change "Proxtexol" to "Protexol."

Again, the committee wishes to express its thanks to Messrs. O'Brien and Reno for their comments, and hopes that the report will be of value to engineers engaged in structural usage of timber.

Respectfully submitted,

T. J. McClellan, Chairman

W. R. Bond
C. F. Craig
F. Gottschalk
A. R. Richards

Sub-Committee of the Committee on Timber Structures of the Structural Division

LATERAL LOAD ANALYSIS OF TWO-COLUMN BENTS[a]

Closure by John E. Goldberg

JOHN E. GOLDBERG,[1] M. ASCE.—The author is indebted to Messrs. Nubar, Sobotka, Chang and Cooke for the interest which they have shown in the preparation of their discussions and for their constructive comments.

Engineers frequently have certain methods of analysis, or approaches to the formulation of methods of analysis, which have become their favorite technique. This favored position may result from early training or from continued practice, very much as the choice of a spoken language or a language of the thought process may result from analogous circumstances, and not necessarily from the fact that the favorite method is significantly better than an alternate available method. If the favored method is valid and not relatively inconvenient, there is of course no particular reason to abandon that method. Among the valid methods, the choice of method is not nearly so important as the fact that the engineer must have valid and practicable methods for handling the most frequent types of problems and, at least one fundamental method which can be applied to the solution of special and unusual problems.

For those who prefer to work in terms of bending moments rather than displacements, Dr. Nubar has suggested an excellent variation of the author's equation and procedure as well as a graphical method for solving a tridiagonal system of linear algebraic equations. It may be mentioned that the variation which has been suggested by Dr. Nubar can be derived directly from Equation (13), which is

$$(6K_n^G + K_n^c + K_o^c)\,\theta_n = \frac{M_n + M_o}{2} + K_n^c\,\theta_m + K_o^c\,\theta_o$$

merely by substituting Equation (11), which is

$$M_n^G = -3K_n^G\,\theta_n$$

thus obtaining

$$\left(6 + \frac{K_n^c + K_o^c}{K_n^G}\right) M_n^G = -3\,\frac{M_n + M_o}{2} + \frac{K_n^c}{K_n^G}\,M_m^G + \frac{K_o^c}{K_o^G}\,M_o^G$$

a. Proc. Paper 1638, May, 1958, by John E. Goldberg.

1. Assoc. Prof., Dept. of Civ. Eng., Purdue Univ., Lafayette, Ind.

The expressions for the column end-moments may be obtained from the original equations by use of the same substitution. As Dr. Nubar states, the convergence of an iterative solution in terms of the moments will be as rapid as a solution in terms of joint rotations.

Dr. Nubar's graphical calculation of the 8-story problem which was used as an illustrative example shows the remarkable accuracy which could be obtained by this simple technique if the set of equations is not too extensive. It may be pointed out that this graphical method corresponds to one of the several step-by-step algebraic procedures which can be used in the solution of a set of equations which have this form. As in the step-by-step algebraic method, considerable care and accuracy must be used if the structure comprises many spans or tiers. The accuracy with which entirely graphical work can be done generally puts a limit on the validity of the results when more than a few spans or tiers are involved.

As Mr. Sobotka's discussion suggests, the classical techniques for solving a set of simultaneous linear algebraic equations may lead to especially simple procedures when the set of equations have certain characteristics. In the present case, the matrix of the coefficients form a tri-diagonal set with triangular blocks of zeros at the upper right and lower left corners. Mr. Sobotka has presented a very useful recursion formula which, taking advantage of this circumstance, greatly simplifies the solution obtainable by means of Cramer's Rule, i.e., the method of determinants.

Mr. Sobotka has also demonstrated a solution, by the methods of difference equations, which can be applied when the stiffnesses of corresponding members of the bent form a geometric progression, including the constant case. The author has found these methods to be interesting and useful in these cases and, in fact, occasionally has taught these methods to his students. Mr. Sobotka's description is exceptionally thorough and lucid.

Mr. Chang has suggested and demonstrated a moment distribution procedure for analyzing symmetrical two-column bents under transverse loads which is a great improvement over the usual shear distribution method. A disadvantage of the basic procedure is its slow convergence, resulting from the fact that the carry-over factor is unity. Mr. Chang, however, has done much to overcome this disadvantage by a procedure which makes use of a block relaxation and is closely related to the so-called precise moment distribution.

Mr. Cooke states that he prefers the cantilever distribution method which Mr. Chang has already mentioned but, unlike Mr. Chang, is willing to accept the slower convergence which results from a carry-over factor numerically equal to unity whereas Mr. Chang has improved the situation somewhat by the use of an intelligent relaxation procedure. If Mr. Cooke, for example, prefers to use a particular method which is technically valid, one cannot argue this point; but Mr. Cooke's statement that this basic form of cantilever distribution is simpler than that presented by the author is not at all consistent with the facts. Mr. Cooke states that <u>ten</u> cycles of cantilever distribution gave moment values, for the eight-story frame used by the author, which were within five percent of the final values. However, employing the iterative method which the author has presented, accuracies better than five percent in all but one moment and six percent at one point are obtained if the rotations given by only <u>one</u> cycle are used, and an accuracy of better than 0.7 percent is obtained using the results of two cycles. Furthermore, the method which has

been presented is to a great extent self-checking, and this is not generally true of distribution methods.

In preparing a paper, an author has at least two alternatives. He may, in a relatively brief paper, present a specific viewpoint, an approach or a method which seems to him to be significant or helpful. He thus seeks, by brevity and concentration, to make an unambiguous and unconfusing presentation which, within established limits, appears to him to be definitive. Alternatively, he may prepare a comprehensive study, frequently becoming academic or pedagogical, in which several viewpoints, approaches or methods are presented. If both possible papers are to have approximately the same length, one must be organized along a vertical line while the other must be organized along horizontal lines.

In this paper, for the sake of brevity, the author has chosen what is essentially the first alternative. This was done with the knowledge that the formulation and methods which have been presented are basic and flexible and will yield results with very nearly a minimum of straightforward effort even under relatively general or irregular conditions. Some of the methods which have been presented could be put on a more formal basis. For example, a specific formula can be deduced readily from Equation (13) for the construction of the equations involving the rotations of the joints at the alternate levels. This, however, is less of a new method than it is simply an exercise, particularly since the algebraic manipulation of Equations (13) after inserting numerical values of the coefficients is so simple.

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DESIGN OF LONG REINFORCED CONCRETE COLUMNS[a]

Discussions by J. G. MacGregor and C. P. Siess, and Phil M. Ferguson

J. G. MacGREGOR,[1] J.M. ASCE and C. P. SIESS,[2] M. ASCE.—The authors are to be commended on the large amount of excellent work represented by their series of three papers on long reinforced concrete columns. The range of the variables studied shows the effect of every practical variable.

Material Failures

If an analysis similar to that described in Ref. A is used to determine the long column strength, the distinction between material failures and instability failures becomes more evident. Figure A shows a family of moment-load curves constructed for hinged-end columns of various ℓ/d ratios and with $e_1/e_2 = +1$. These curves relate axial load at any stage of loading to the maximum moment occurring in the column at the same load (at the section of maximum deflection). If this curve passes through a maximum, the column under consideration will fail initially by inelastic instability which in turn leads to material failure as shown by the dashed lines. If the moment-load curve intersects the interaction curve before reaching a maximum, as it does for $\ell/d = 5$, the column will fail by material failures. The long column curves obtained by this method agree with those published in the authors' Fig. 3.

Column Tests

In Ref. B, Tables 2 and 3, ratios of test strength to calculated strength are reported for 185 eccentrically loaded columns. The calculated strengths are based on the actual failure eccentricity at the section of failure. The average ratio of test to calculated strength using the final eccentricity for all 185 columns is 0.98; however, when the strength is computed on the basis of initial eccentricity, it is only 0.94 for the 114 columns tested by Hognestad and only 0.89 for all 185 columns reported. This emphasizes the fact that there is a reduction in the strength of columns with an ℓ/d as small as 7.5.

Design Equations for Long Columns

In ignoring end restraints in deriving a design equation, the authors are being justifiably conservative. In addition to the reasons listed by the authors,

a. Proc. Paper 1694, July, 1958, by Bengt Broms and Ivan M. Viest.
1. Research Asst., Dept. of Civ. Eng., Univ. of Illinois, Urbana, Ill.
2. Prof. of Civ. Eng., Univ. of Illinois, Urbana, Ill.

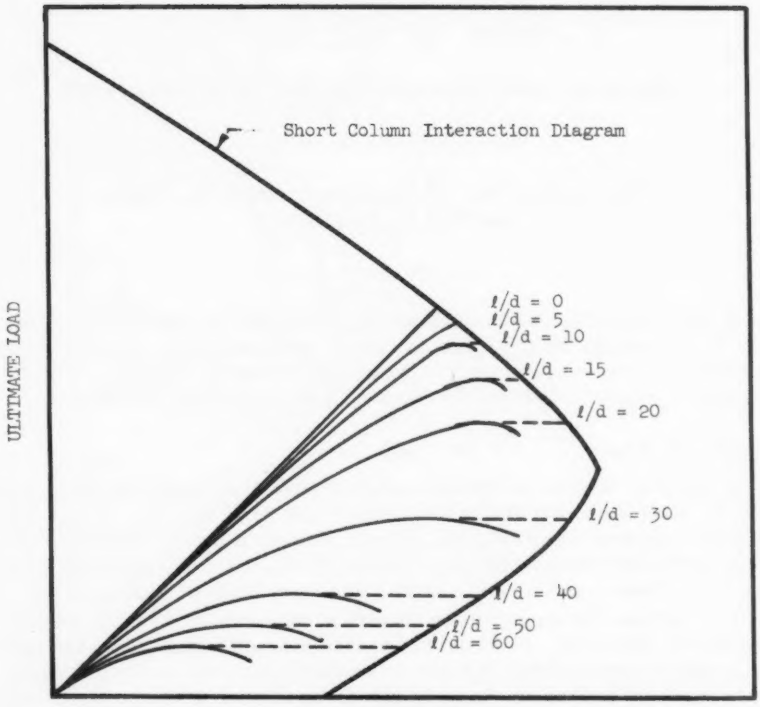

Fig. A - MOMENT LOAD CURVES FOR ECCENTRICALLY LOADED COLUMN OF A GIVEN INITIAL ECCENTRICITY ($e/d = 0.2$)

the following points also support this stand. The restrained column analysis developed in ASCE Proc. Paper 1635 takes into account all reductions in column stiffness under loading but assumes that the stiffness of the restraining members remains constant throughout the life of the column. In tests of rigid frames reported by Richart,(C) the EI of a flexural member at ultimate load under short-time loadings was about 0.4 times the EI of the uncracked section at working loads. This reduction was caused by cracking, plastic action in the compression zone, etc. Creep from sustained loading in the restraining members would further reduce their stiffness as would yielding of the reinforcement in these members. Each reduction in the stiffness of the restraining members brings the column action closer to that of a hinged-end column.

In developing a column design formula, the loading conditions expected in structures should be considered. Briefly, three types of column failures can be distinguished. First, the load and moment may be of short duration as is the case with wind loads. Second, and perhaps most common in practice, the column will be loaded to a certain percentage of its capacity under a sustained

deadload, and then more or less rapidly loaded to failure. Third, the total applied load or moment also may be of long duration as in a warehouse.

The first loading condition corresponds to the short-time load analysis and also corresponds to the type of loading employed in the column tests used to check the validity of the authors' design equations. The close check between the short-time load analysis for hinged-end columns and these column tests establishes the validity of the analysis used.

The second loading condition corresponds approximately to the assumptions made in the authors' long-time analysis. It has been assumed that sustained loadings cause the strains to double but do not affect the strength of the concrete in the column. This is the case for a column loaded with a sustained working load which is rapidly increased until the column fails, as was shown in the tests reported by Viest, Elstner and Hognestad.(D)

A design equation for long columns should apply basically to the case of a sustained working load rapidly increased until failure occurs but, when necessary, should be modified to take account of failure under sustained high loads. The validity of such a design formula can be checked only by comparing it to the results of tests on long columns loaded in this way. The writers know of no tests of eccentrically loaded long columns which involved sustained loading. A possible exception would be the column tests described by Viest, Elstner and Hognestad in which the columns had an ℓ/d of about 7.5 including the end blocks. Therefore, it is not possible to determine the accuracy of a proposed long-column design formula by comparing it to test results as has been done by the authors. Since the short-time load analysis checks closely with available test results and since the assumptions made to extend this analysis are reasonable and are based on experimental studies, the writers have accepted the long-time analysis in the authors' first paper as a valid analysis for the type of column loadings experienced in structures. Accordingly, the following reduction equation for long eccentrically loaded columns failing in compression under the usual types of loadings is proposed:

$$R = 1.20 - 0.025\ \ell/d - 0.13\frac{e_1}{e_2} \leq 1 \qquad \text{(A)}$$

This equation was derived by fitting a line to the long-time load analysis shown in Fig. 3(f). It was also compared to an analysis made as mentioned above and based on the assumption that the ratio of total sustained-load strain to elastic strain is three. Equation A and the curve from Broms and Viest's long-time load analysis are plotted in Fig. B. For simplicity, only the curves for $e_1/e_2 = +1$ have been plotted in this figure. The close agreement between Eq. A and the curve from the long-time load analysis can be seen clearly.

Equation 2, proposed by the authors, is also plotted in Fig. B. This equation does not represent the behavior of a long column. The authors also recognized this and modified their equation by multiplying it by a factor of 0.84, representing the theoretical long-column strength reduction for $\ell/d = 10$. The resulting equation (Eq. 2 times 0.84) is:

$$R = 1.218 - 0.0252\ \ell/d - 0.126\frac{e_1}{e_2} \leq 0.84 \qquad \text{(B)}$$

Equation B is also plotted in Fig. B. For ℓ/d greater than 10, Eq. B is almost identical with Eq. A. As can be seen from Fig. B and Eqs. A and B, the multiplying factor of 0.84 is actually an integral part of the long-column

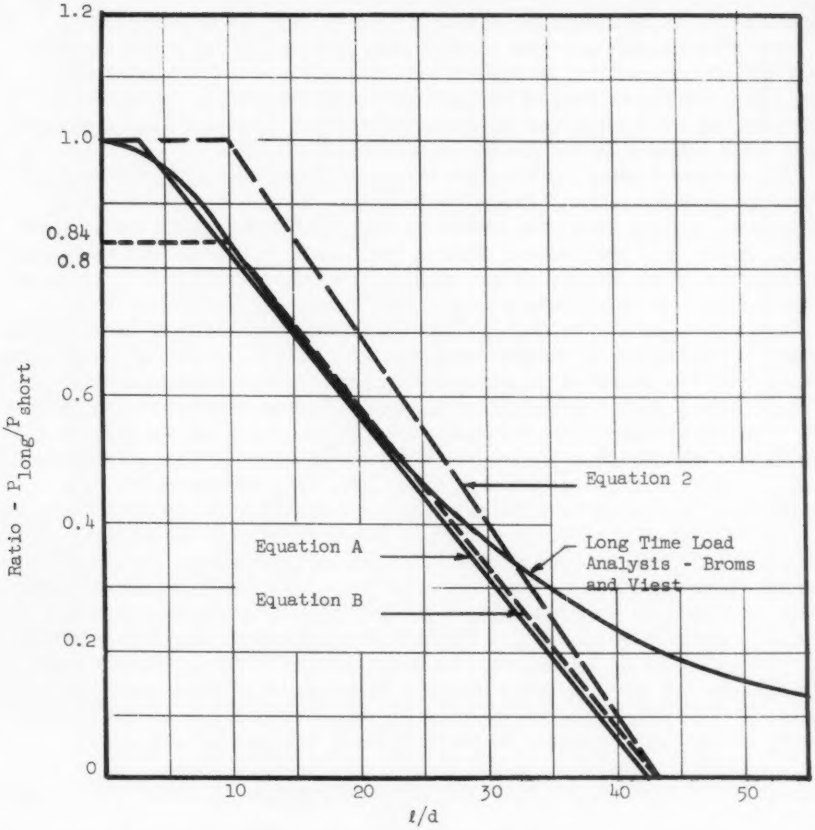

Fig. B - COMPARISON OF LONG COLUMN EQUATIONS FOR e_1/e_2 = +1

reduction formula, and as such we believe it should be included in the equation.

Because the 0.84 term is actually a part of the required long-column formula, it cannot be considered as a part of the load factor. Thus, the actual load factor for columns that the authors propose for use in Eqs. 5(a) and 5(b) is K = 1.1, not 1.3 as stated in their paper. The use of Eq. 2 and the proposed load factor for columns leads one to the belief that column design is considerably more conservative than actually is the case.

The form of Eq. B, that is, the upper limit at 0.84, tends to penalize short columns. Thus, columns of ℓ/d = 7, 8 and 9 are required to have safety factors equal to 103, 106 and 110 percent, respectively, of the safety factor for a column with ℓ/d = 20. These factors do not reflect the relative safety of such columns or the relative possibilities of construction errors such as honey-combing. Since columns with ℓ/d ratios less than 10 are common in modern construction, especially with the trend to higher concrete buildings, it does not seem logical to limit the strength of columns with ℓ/d less than 10, to preserve an equation which by itself is not correct.

Modification for Sustained High Loads

Tests by Shank[(E)] and Rüsch[(F)] have shown that the strength of concrete cylinders and prisms subjected to sustained high loads is reduced by 10 to 30 percent. This strength reduction must be considered in the design of columns subjected to sustained loads to failure (the third loading condition). Thus, for this loading condition, the short-column strength can be approximated by using a reduced concrete strength equal to $0.85\ f_c'$ in the computations to allow for the reduction in concrete strength under sustained high loads. This reduction is in addition to the commonly accepted $0.85\ f_c'$ factor (Assumption 4,[(B)] used to correlate the strength of concrete in flexure with the strength of a standard 6- by 12-in. cylinder.

The term "sustained load" must also be defined. Several foreign codes specify that a load of three days duration or longer should be considered a long-time load. This appears to be a reasonable value. In the above mentioned tests by Rüsch, concrete loaded to 80 percent of its short-time strength failed in about four days, that loaded to 75 percent of its short-time strength failed in seventy days.

Tension Failures

For eccentrically loaded columns failing in tension, the actual length reduction factor is considerably smaller than that required for compression failures (see, for example, the curve for $e_2/d = 0.8$ in Fig. 3 c). The limiting case is a beam loaded in pure moment with no axial load. In such a beam there is no ℓ/d effect. For this reason, Eq. A should be limited to compression failures and the following reduction equation is proposed for the tension failure region:

$$M' = M_o + (RM_b - M_o)\frac{e_b}{e} \tag{C}$$

where

- M' is the desired reduced moment for a long column;
- M_o is the moment capacity at $P = 0$;
- M_b is the moment capacity at a balanced tension-compression failure in a short column;
- e_b is the eccentricity corresponding to M_b for a short column;
- R is the long-column reduction factor from Eq. A.

Figure C compares the various proposed reduction equations and the theoretical analysis by the authors. If design is carried out using an interaction diagram, Eq. C could be replaced with a straight line joining RM_b and M_o. This matter is not just of academic importance since the lower portions of the interaction curve apply to many situations where large moments combine with relatively small axial forces, such as in the girders of rigid frames.

Shape Effects

The authors' analysis was derived using a square column with steel concentrated in two faces. Strictly speaking, the long-column curves obtained

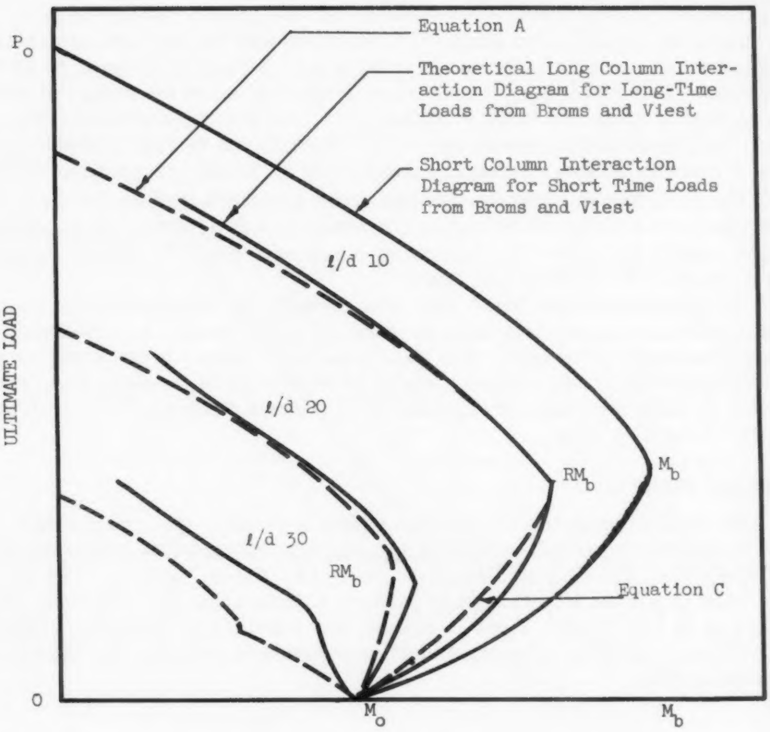

Fig. C - COMPARISON OF LONG COLUMN FORMULAS AND THEORETICAL INTERACTION DIAGRAMS FOR LONG COLUMNS

apply only to columns of this cross-section. From the column tests quoted above(B) we see that circular columns of $\ell/d = 7.5$ tested by Hognestad had a mean long-column strength ratio of 0.93 while the rectangular columns tested by Hognestad had a strength ratio of 0.955. The effect of various cross-sections on the buckling strength of concrete columns is also indicated in the discussion by Hromadik of Proc. Paper 1510. The radius of gyration of a circular reinforced concrete column ranges from about 0.26d to 0.30d while that of a rectangular reinforced concrete column ranges from about 0.29d to 0.35d. If the curves for a square column and a circular column in Hromadik's discussion are replotted in terms of ℓ/r they reduce almost to one curve. This suggests that the long-column reduction equation should be written in terms of ℓ/r rather than ℓ/d, or alternatively, that a different reduction equation should be used for each type of cross-section. Since the ℓ/r of a circular column is between 105 and 125 percent of that of a square column of the same ℓ/d, the error involved in using d instead of r might be serious for the longer columns.

Miscellaneous

The writers would like to have the authors state which e_1/e_2 ratio should be used in the design of long "centrically loaded" columns, along with the reasons for their choice.

SUMMARY

1) The authors' short-time and long-time load analyses are valid for the column shapes considered. The decision to neglect the beneficial effects of end restraint in a long column design equation is conservative, but is well grounded.

2) A design equation for long columns should apply basically to the case of a sustained working load rapidly increased until failure occurs, but when necessary should be modified to take account of failure under sustained high loads.

3) The validity of a column design formula can be determined only by comparing it to tests of columns loaded under sustained loads. If comparisons are made on the basis of long-column tests now available, all of which involve only short-time loadings, the design formula under consideration will appear to have a greater factor of safety than it actually has.

4) The following long-column design equations are proposed. These apply to all eccentrically loaded columns designed on the basis of initial eccentricity.

a) For eccentrically loaded columns failing in compression under a sustained working load which is rapidly increased until failure occurs, the length effect should be taken into account by the following equation:

$$R = 1.20 - 0.025\, l/d - 0.15 \frac{e_1}{e_2} \leq 1 \qquad \text{(A)}$$

b) For eccentrically loaded columns failing in compression under sustained high loads, lasting more than three days, the length effect should be taken into account by using a reduced concrete strength equal to $0.85\, f'_c$ in computing the short-column strength.

c) For case (a) or (b), with eccentrically loaded columns failing in tension, the length effect should be taken into account by the following equation:

$$M' = M_o + (RM_b - M_o) \frac{e_b}{e} \qquad \text{(C)}$$

where M_o, M_b, and e_b are computed for the short column using the appropriate concrete strength for the loading involved.

REFERENCES

A. Clark, M. E., Sidebottom, O. M. and Shreeves, R. W., "Inelastic Analysis of Eccentrically Loaded Columns," Proc. ASCE, Vol. 83, Oct. 1957, No. 1418.

B. "Report of ASCE-ACI Joint Committee on Ultimate Strength Design," Proc. ASCE, Vol. 81, Oct. 1955, No. 809.

C. Richart, F. E., "Tests of the Effect of Brackets in Reinforced Concrete Rigid Frames," Bureau of Standards Journal of Research, Vol. 1, No. 2, Aug. 1928, pp. 189-250.

D. Viest, I. M., Elstner, R. and Hognestad, E., "Sustained Load Strength of Eccentrically Loaded Short Reinforced Concrete Columns," ACI Journal, Proc. Vol. 52, March 1956, pp. 727-755.

E. Shank, J. R., "Plastic Flow of Concrete at High Overloads," ACI Journal, Proc. Vol. 45, Feb. 1949, p. 493.

F. Rüsch, H., "Investigations into the Strength of Concrete under Sustained Load," RILEM Colloquium, Munich, Nov. 1958.

PHIL M. FERGUSON,[1] M. ASCE.—Much valuable information is contained in this paper and in the authors' two preceding papers(1,2) on this general subject. The analysis of reinforced concrete columns as long columns, taking into account both the case of "elastic" buckling and material failures which occur under smaller lateral deflections, is a considerable accomplishment. Nothing in this discussion questions these theoretical analyses.

However, columns as a portion of a frame are much influenced in their behavior by the action of the frame as a whole. This interaction between columns and other frame members is much more varied than can be represented by any single restraint mechanism.

The authors correctly point out that the "ultimate strength of restrained columns is always higher than the strength of hinged columns--." They then recommend the use of a reduction coefficient for all long columns and base this reduction solely upon the strength of hinged columns, with no recognition given to the higher strength available with restrained columns in a frame. Since the strength differences between long hinged columns and long restrained columns are major and not minor differences, these design recommendations appear to be economically unsound for a large percentage of all column designs. It is suspected that this percentage might be from 80 to 90 percent.

The authors seem not to have differentiated in their analyses between columns subject to direct loads which are essentially matters of statics and those moment loads which are impressed by angle changes and hence are deformation type moments. This distinction is important and must be incorporated before a column study can lead to realistic design specifications.

For this discussion columns will be considered under several groupings:

1. Columns where sidesway effects may be ignored.
   a. Interior columns with similar eccentricities at each end as in Fig. Aa.
   b. Exterior columns in reversed bending as in Fig. Ab.
2. Columns where sidesway is important.
   a. One story rigid frames as in Fig. Ac.
   b. Buildings where lateral wind loads produce important column shears and moments as in Fig. Ad.

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1. Professor of Civil Engineering, University of Texas, Austin, Texas.

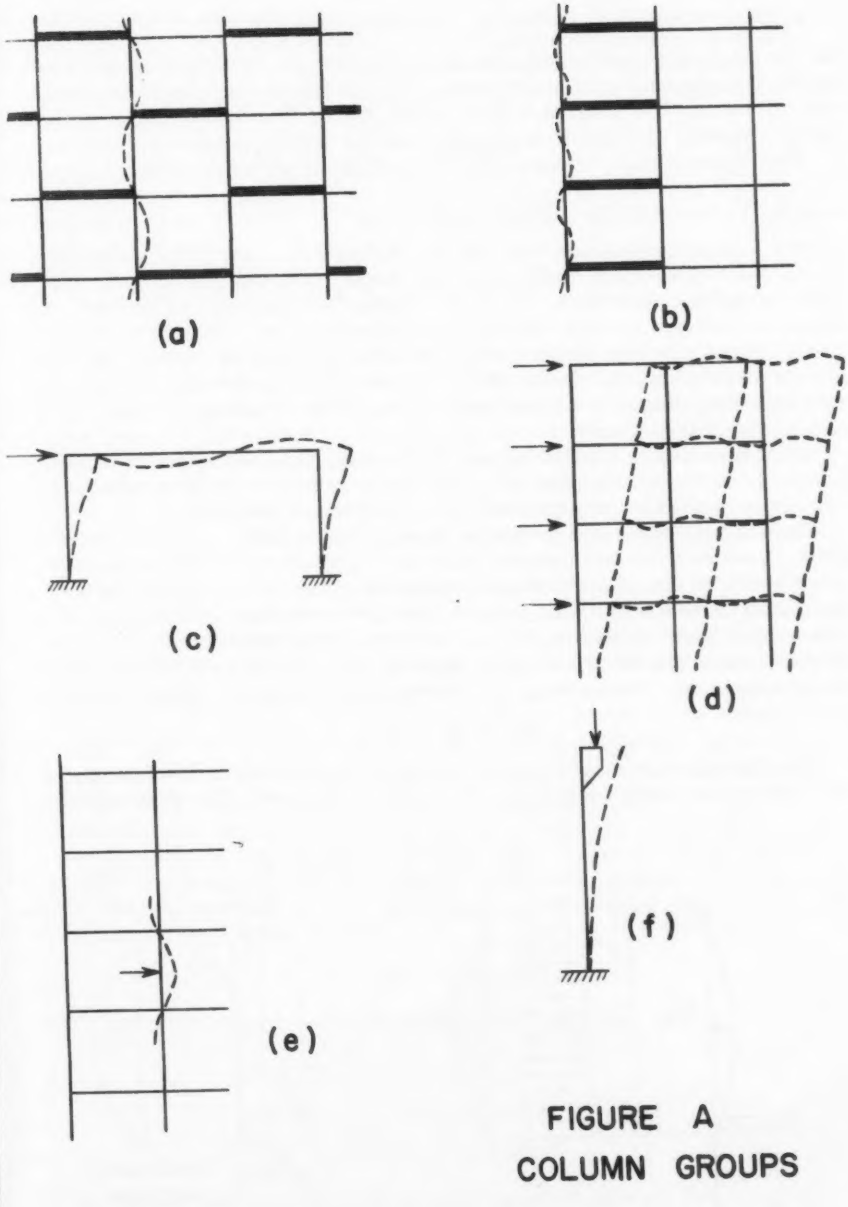

FIGURE A
COLUMN GROUPS

3. Columns directly loaded between joints as in Fig. Ae.

4. Free standing, "flagpole" type columns as in Fig. Af.

By far the largest number of columns fall in group 1 and this discussion is primarily related to this large group. Limitations of time, amongst other commitments, have prevented an adequate study of the columns in group 2; and the number of columns in groups 3 and 4 are quite small.

This discussion is directed entirely to ultimate strength design.

Interior Columns Without Sidesway (Fig. Aa)

The authors recommend that the end moments on columns be established on the basis of an elastic analysis of the frame; there is now no adequate alternate to this procedure. However, it should be emphasized that columns (without sidesway) usually receive their moment loads only from joint rotations. These moments are impressed or deformation type moments and are far from being statically determined; any change in joint rotation causes a corresponding change in column moment and in the resulting eccentricity $\underline{e}$ of the load on the column.

While this situation might be more accurately analyzed by the use of differential equations, this discussion will be presented in the simpler terms ordinarily related to moment distribution and frame analysis.

The analysis of the column in Fig. B will first be based on elastic conditions. Assume equal and opposite moments at the two ends, which are indicated as Pe in the adjacent moment diagram. This moment produces an angular deflection at the end, or joint, which is consistent with the joint rotation used in frame analysis. Because the column deflects when carrying moment, there will be an additional moment $P\delta_m$ at the center of column, which adds to Pe. The critical column moment is thus the center moment:

$$M_c = Pe + P\delta_m$$

This deflection moment exists in some measure all along the column, as indicated by the second segment of the moment diagram. The shape of this

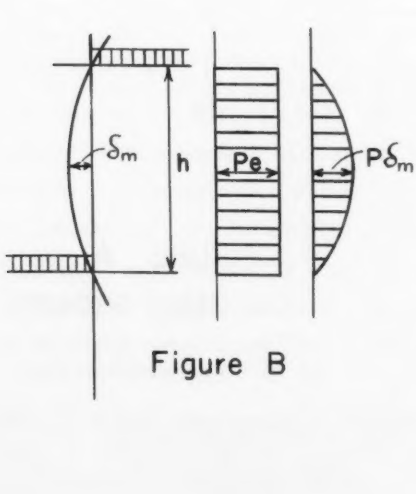

Figure B

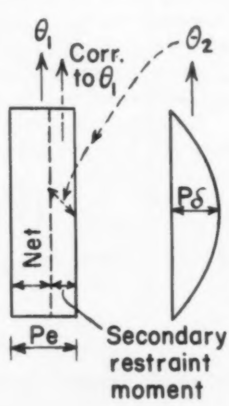

Figure C

segment is determined by the shape of the deflected column. Although it would be interesting to establish the equation for the elastic curve of the column, such accuracy is unnecessary for the intended use here. It will be adequate to say that the curve is a circular segment or a parabola or a cosine curve. As such, the average moment value may be arbitrarily and approximately assumed as $0.64\ P\delta_m$.

The usual methods of frame analysis are based on member stiffness which is the member resistance to rotation at a joint. The rectangular moment area Peh indicates an angle of rotation of the column at each joint which, from symmetry must be $Peh/(2E_cI_c)$; and this alone would make the end slope of the column fit perfectly with the joint rotation used in frame analysis. However, if an additional moment $P\delta_m$ exists, as it must, the angle of rotation at the end of the column must be increased by θ_2 = half the area of the M/EI diagram $= 0.64P\delta_m h/(2E_cI_c)$.

This extra angle might be interpreted in two roughly equivalent ways. One might say that a column carrying moment is further deflected by the secondary moment $P\delta_m$; therefore such a column is less stiff against end rotation. If this is the case the moment distribution factors for the joints should be revised in such a manner as to reflect this lower column stiffness. The lesser stiffness would result in a smaller moment distributed to the end of the column, that is, smaller than the original Pe. The center column moment would then be less than $Pe + P\delta_m$; and δ_m itself would also be reduced, because of the smaller moments acting.

Instead of the above, one might first say that if the moment Pe were statically determined and the joints were hinged, the column would be entirely free to rotate through this added angle θ_2 and the maximum column moment would indeed be $Pe + P\delta_m$. Next, since the joint is not actually hinged, consider the change in moments which would develop around the joint if the joint did rotate through this full extra angle θ_2. As the extreme case, assume that the column above the joint also tends to develop this same extra angle θ_2 and that beams are symmetrical. This in effect would lead to the statement that each column would pull one of the beams through the angle θ_2. Each beam would resist such rotation with a moment of approximately $4E_bI_b\,\theta_2/L$; and in order to maintain joint equilibrium this beam moment would have to be balanced at the joint by an equal change in the column moment. This balancing moment would represent a reduction in the original moment Pe such that the end column moment would become

$$Pe - 4E_bI_b\theta_2/L = Pe - 4E_bK_b\theta_2$$

and the maximum column moment at midheight would be

$$\begin{aligned} M_c &= Pe + P\delta_m - 4E_bK_b\theta_2 \\ &= Pe + P\delta_m - 4E_bK_b(0.64P\delta_m h/2E_cI_c) \\ &= Pe + P\delta_m\left(1 - 1.28\frac{E_bK_bh}{E_cI_c}\right) \end{aligned}$$

This equation indicates that the original assumed moment $Pe + P\delta_m$ is larger than that which could accompany δ_m and the extra θ_2, and this regardless of how small δ_m may have been assumed. It should also be noted that this smaller moment would imply a smaller θ_2, but that the smaller θ_2 would

simply reduce the negative term and not change the form of the equation. It thus becomes obvious that the column design may safely be based on a moment smaller than the sum of the moment Pe obtained from an elastic analysis and the moment $P\delta_m$.

The parenthesis in the equation for M_c should also be examined as to its absolute value. If δ_m were initially assumed at its final correct value, the final angle changes could be shown diagramatically as in Fig. C. The moment Pe and the angle θ_1 would be known from frame analysis. The angle θ_2 would result from the deflection moment represented at mid-height by $P\delta_m$. The angle θ_2 would develop the secondary restraining moment represented by the negative term in the equation for M_c. This restraining moment would in turn produce a negative angle change which might just as accurately be considered as a reduction in θ_1. With the use of the correct assumed δ_m, the equation for M_c would then be exact, within the elastic range of the materials, except for errors in the assumed shape of the deflection curve. Such errors could only have the effect of modifying the average deflection ordinate to something other than $0.64\delta_m$ and of modifying the constant of 1.28. Since it is difficult to imagine a column shape that would have an average deflection ordinate less than $0.5\delta_m$, the 1.28 constant might range between 1.00 and 1.30.

The parenthesis in the equation for M_c becomes zero when $1.28\ E_bK_bh/E_cI_c = 1$; or, if the conservative value of 1.00 is used instead of 1.28, when $E_bK_b = E_cI_c/h$. Thus if the beam is at least as stiff as the column, the final moment on the column will not exceed Pe as obtained from elastic analysis of the frame. If the beam is the stiffer member, the column design moment will be less than Pe. Thus in a frame, the longer and smaller such a column is made, the less probable it becomes that the "initial" end moment Pe can influence the buckling of the column. The joint restraint can even be large enough to reverse the sign of the end moment on the column in the case of a very slender column. A design based simply on the frame moment Pe will thus be a conservative way of preventing failure of such columns.

It is noted that should the column stresses approach failure and thus change the column into one which is less stiff, this effect can be represented in the formula for M_c by using a lower value of E_c. Thus, before failure, the column stiffness would be further reduced relative to that of the beam; and M_c would become still smaller, with Pe replaced by a resultant end moment of opposite sign.

In the case of a very stiff column, where column stiffness is twice that of the beam, δ_m will be numerically small and the effect of the resulting $P\delta_m$ term is reduced at least 50% by the negative term representing the frame action. If in addition the value of E_c decreases, under overstress, by as much as 50 percent, this reduction in the $P\delta$ moment becomes at least 75 percent. It is strongly suspected in this case that the proper constant in the equation for M_c should be closer to 1.28 than to 1.00. In such case these reductions become about 64 percent and 95 percent, respectively.

This discussor thus concludes that, except for columns extremely stiff with respect to the floor system, it will be amply safe in ultimate strength design to proportion for the moment Pe obtained from elastic frame analysis with no addition for deflection moment or length effects. As a restriction against very long columns, the proviso might be added that such columns must not be smaller than the hinged column needed to carry the axial load with zero

eccentricity. This conclusion applies only to interior columns without significant sidesway.

Exterior Columns in Reversed Bending

A column such as that in Fig. Ab may be considered as loaded in moment by equal angle changes at each end. In such a case the maximum moments from frame action occur at the ends of the columns and the maximum deflection moments are nearer the quarter or fifth points of the column. Thus deflection moments are less likely to create total moments in excess of the end moments.

Compared to the previous analysis of the interior column, this case has only one beam to restrain the two columns (one above and one below) but the deflection effect is so very much smaller that further analysis seems unnecessary. The authors' Fig. 3e shows a curve for a specific column having $e_1/e_2 = -1$, which is intended to represent this case. Although this curve shows no reduction in strength until L/d is 30, it is unduly conservative because it is based on hinged ends and does not recognize the effective restraint provided by the frame.

It appears that ultimate strength design for these columns based on the Pe moments calculated from elastic frame analysis is quite safe and that no recognition needs to be given to deflection problems. It might appear that a moment applied at only one end of the column might produce more of a deflection problem. However, this case could not be as serious a problem as that of the interior column already discussed. Since the reversed bending moment Pe will be larger, it is the governing case.

Columns With Important Sidesway (Figs. Ac and d)

When sidesway is important, especially where lateral loads such as wind or earthquake forces are a significant part of the loading an entirely different problem exists. The column shears and moments in this case are more a matter of statics than a matter of impressed moments caused by end rotations. In other words, Pe moments calculated for such columns remain undiminished as joints rotate; hence deflection moments are directly additive. Time has prevented this discussor from carrying out a more thorough analysis of such columns. It appears that the authors' hinged column assumption is desirable here. However, it would appear that each half of the column approximates half of a column loaded as in Fig. Aa, which would mean that $e_1/e_2 = +1$ would be more correct for use in their equations than $e_1/e_2 = -1$.

Columns Directly Loaded Between Joints (Fig. Ae)

The authors' analysis of restrained columns appears more directly applicable to this case than to columns loaded through connecting beams. This is a loading which can be measured chiefly in terms of statics and the authors' analysis is based on a static moment Pe. For a proper understanding of the entire column problem these predominantly static moments must be contrasted with moments impressed by angle changes. The authors' method appears poorly adapted to the latter case, as in the interior columns first discussed.

Free Standing, "Flagpole" Type Columns (Fig. Af)

There are only a small number of cantilever type columns and these include those crane columns which are not braced across the crane bay. If these columns are actually fixed at their bases, their curvature is exactly like that in the upper half of a hinged end column which is similarly loaded at each end. Thus, as for any other material, design must be based on the use of an effective length twice the actual column length. Under this condition the hinged end column analysis is exactly correct.

The Authors' 0.84 Factor to Account for Deflection

This discussion has in a large measure been concerned with deflection moment and the counterbalancing effect of frame restraint. For interior and exterior columns it has been shown that deflection moments are generally not important, since frame restraint reduces or cancels the apparent ill effects. There thus seems to be no need for a reduction factor of 0.84 for these cases, which constitute the majority of all columns.

If the 0.84 factor is needed for some cases, and it is not yet established clearly where this would be, it should certainly not be written into the load factor for all columns. Neither would it appear desirable to apply it to increase the design axial loads. It should be directly related to the moments involved and then only for the particular columns where it is needed. Thus the proposal that the factor K in the authors' suggested load factors be made 1.3 is totally unacceptable to this discussor.

Effect of Plastic Hinges

The authors' consideration of the effects of plastic hinges in restraining members seems to provide one of the reasons they recommend that the strength of hinged columns be used for all designs. For the interior and exterior columns (without sidesway) which have been the primary reason for this discussion, it appears that the restraint moments discussed are all in the reverse sense to those which might create hinges. Thus the restraints which this analysis has considered are available for some range even after the formation of plastic hinges. This available resistance assumes the beam has also been designed for maximum moments based on elastic frame analysis. If such beams should (at some future date) be designed by limit design procedures, it might be necessary to consider deflection-caused moment from long columns in the beam design in order to avoid failure in the beam. However, limit design is not yet an approved design method in reinforced concrete in the U.S.A. By the time limit design is adopted, it is hoped that much more comprehensive studies of columns will also have defined more exactly the associated problems of long slender columns.

BIBLIOGRAPHY

1. B. Broms and I. M. Viest, "Ultimate Strength Analysis of Long Hinged Reinforced Concrete Columns," ASCE Proceedings, Paper No. 1510, Jan., 1958.

2. B. Broms and I. M. Viest, "Ultimate Strength Analysis of Long Restrained Reinforced Concrete Columns," ASCE Proceedings, Paper No. 1635, May, 1958.

WIND FORCES ON STRUCTURES: FUNDAMENTAL CONSIDERATIONS[a]

Discussion by John S. McNown and Dorris Hankins

JOHN S. McNOWN,[1] M. ASCE and DORRIS HANKINS[2].—Evaluation of the forces exerted on structures by wind requires an understanding of several basic principles of fluid mechanics. These include the concept of the Reynolds number, the role of viscosity in causing separation, the resulting distribution of pressure over a body subjected to wind, the consequences of periodic or otherwise unsteady occurrences, and, in extremely high winds, the Mach number. These points are introduced by Messers. Woodruff and Kozak, but their presentation is rather incomplete. Also, their terminology and interpretations differ in several instances from those generally accepted in the fields of fluid mechanics and aeronautics with a resulting lack of consistency and clarity.

Terminology

In the development of fluid mechanics, the terminology has been progressively refined. Such misnomers as "velocity pressure" and "aerostatic wind forces" have been rejected as inconsistent and misleading. The increase in pressure attributable to the partial or complete retardation of a fluid is known as "dynamic pressure," to distinguish it from pressures attributable to the static (barometric) pressure due to the weight of the fluid. The terms "aerostatic" and "wind" are contradictory. To designate air in motion, one uses the term "aerodynamic" (and for water in motion - "hydrodynamic"). A distinction must also be made between "pressure" (force per unit area) and "force" (a pressure integrated over an area). Because "pressure" is usually related to atmospheric pressure, it can be positive or negative. As a consequence, "pressure" and "suction" are not necessarily in contrast. In fact, the term "suction" has properly given way to "negative relative pressure."

The distinction between "aerostatic" and "aerodynamic" made by the authors is conventionally made by designating the former as "steady" (not changing with time) and "unsteady." The usage is obviously different from that used by structural engineers in designating "dead," "live," and "dynamic" loads. However, if the results of developments in fluid mechanics are to be utilized without confusion, the terminology must be correctly utilized, also. For these reasons, several of the comments on pp. 1, 2, 7, and 9 tend to promulgate a usage which is less preferable because it is in conflict with established practice and because it is less accurate.

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a. Proc. Paper 1709, July, 1958, by Glenn B. Woodruff and John J. Kozak.

1. Dean, School of Eng. & Archt., Univ. of Kansas, Lawrence, Kansas. Also, Consultant, Sandia Corp., Sandia Base, Albuquerque, N. Mexico.
2. Physicist, Nuclear Burst Dept., Sandia Corp., Albuquerque, N. Mexico.

Cause of Drag

As stated by the authors, reduction in velocity, or better reduction in momentum, is the cause of drag. This is as true for drag associated with surface shear (often called "friction") as it is for drag associated with separation, wake formation, and the consequent differences in pressure on the upstream and downstream faces. No drag occurs for potential fluid flow because the pressure distributions in the two directions cancel each other. Only if the effects of viscosity are included is a drag explainable. Either through shear at the surface of the body (with a component in the direction of flow, of course) or through separation induced by the boundary layer, viscosity plays the dominant role. Thus, the Reynolds number is the criterion for similarity. The greater the Reynolds number the more the inertia of the flowing fluid becomes dominant in comparison to the shearing stresses.

If the wind velocity is high enough the effects of compressibility as reflected by the Mach number must be included. For moderate Mach numbers (less than about 0.8) the formula

$$P_{(stag)} = P_{(static)} + q\left(1 + \frac{M^2}{4} + \frac{M^4}{40} \ldots\right)$$

is applicable. Thus, for a Mach number of 0.2 (a velocity for standard air conditions of 230 fps), the correction is only 1%. Obviously, no correction is needed for ordinary winds. Nevertheless, the winds created by a nuclear explosion are strong enough that effects of compressibility cannot be ignored. Extensive studies of various factors, including Mach number, have been presented in a report having limited circulation.(1)

Because the shapes of structures or of structural members are usually bluff rather than streamlined, form drag is more important than surface drag. Losses of kinetic energy in the boundary layer, although a localized phenomenon, cause separation and the creation of a large region in which the pressure is low. The effective boundary of the flow in this region is not the body, and the flow loses the possibility of closing smoothly with a recovery of the positive pressure which would be obtained if energy were fully conserved. Separation occurs at a point of low pressure, and the pressure in the wake is approximately equal to this low value; the total drag is the resultant of increased pressures on the upstream side and reduced pressures on the downstream side.

The distribution of pressure around any body is continuous even at the sharp edge of a thin plate like the one in the authors' Fig. 1 (d). A discontinuity in pressure would imply infinite pressure gradients and infinite accelerations. Hence, the diagram of pressure distribution in the upper part of the figure is incorrect. Also, the maximum (stagnation) pressure does not occur at the edge of an inclined plate as stated near the bottom of p. 3, but rather at a point which approaches the edge more and more closely as the plate is turned so as to be more nearly parallel to the flow. Even for flow which separates, this position can be approximated by means of the free-streamline analysis.(2) For a plate turned 40^o from the normal position, the stagnation pressure occurs at a point about one-tenth of the plate breadth from the edge. From this point, the pressure falls rapidly to the value on the downstream side. Hence, the two curves (for upstream and downstream pressure) should meet at both edges.

Asymmetry does, as the authors state, usually result in a moment. Indication of this moment by means of the formula,

$$M = C_m q A,$$

is both misleading and impractical. A coefficient with the dimensions of a length is not truly a coefficient. The value to be assigned to it will depend directly on the size of the plate. The coefficient e in the other part of the equation,

$$M = e h q A,$$

is properly dimensionless and invariant with size, but it does not contain any indication of the variation of drag or moment with shape, orientation, etc. A proper equation must have the form

$$M = C_D e h q A$$

in which h is the dimension of the plate in the direction in question.

Unsteady Flow

Periodic disturbances caused by the shedding of vortices are the commonest type of wind-induced unsteady loading. Other types are attributable to true unsteadiness in the flow, such as the gusts mentioned by R. H. Sherlock (in Proc. Paper 1708 of the same series) or as the rapidly decelerating flows following a blast. The fluid mechanics of steady flow is sufficiently complicated that as yet little has been done to increase our understanding of drag in a very unsteady flow.

Two recent studies, one related to wave forces(3) and the other to the effects of blasts,(4) contain some information on the effects of unsteadiness on flow past bluff bodies. Three intervals of time must be considered, the time for a vortex to grow to the point of being shed, the time for a pressure wave to travel a distance characteristic of the size of the body (for compressible flow), and a time which is characteristic of the unsteady occurrence. The relative magnitudes of these times help answer such questions as whether (a) the flow is effectively incompressible, (b) separation has time to form, or (c) the flow is quasi-steady.

As the first vortex forms, the characteristics of the flow are markedly different from those for an otherwise comparable steady flow. The coefficient of drag may increase several-fold. In addition, the pressure gradient causing the acceleration and the virtual mass of the fluid (also wake-dependent) may affect or even dominate the total resistance. If effects of compressibility must also be added, the problem becomes so complex that only rough predictions can be made.

Corrections

The first p on both sides of the first equation (p. 2) should be ρ, the mass of the fluid per unit volume.

The symbol q should be defined as $1/2\ \rho\ v^2$.

Size has no intrinsic effect on drag (near the bottom of p. 2) except as it enters into the Reynolds number.

Values of the drag for plates mounted on a plane representing the ground depend upon the degree of development of the boundary upstream from the plate.(5) The pressure distribution is not usually like that shown in Fig. 1 (c).

An eddy forms at the base of the plate and prevents stagnation pressure from occurring. Consequently the value for a flat plate on a ground plane varies considerably from test to test.

The statement near the bottom of p. 3 that "The suction is higher at the leading edge than at the trailing edge" is not supported by the authors' reference 4. For an infinite plate the two are essentially equal for angles between 0 and 60°.

The heading for Fig. 1 (a) is misleading because the figure contains values for $\lambda = 1$. The values for this case in Figs. 1 (a) and (b) are not consistent. Other authors give values ranging from 1.11 to 1.18. Also, for $\lambda = 1$, the positive pressure on the upstream side is much greater than the negative pressure on the downstream side. In both places, the authors have apparently interchanged the two values.

On p. 7, unequals (to the extent of a factor of 1/2) are equated.

The existence of a vortex trail is substantiated only for cylindrical bodies. The upper limit of the Reynolds number for vortices to occur is not well defined - the authors indicate it to be 100,000 at one point, 200,000 at others. The governing Strouhal number is also a function of the Reynolds number.(6) An excellent summary of this problem has been presented by Rosenhead.(7)

REFERENCES

1. Hankins, D., "Experimental Pressure Distributions and Force Coefficients on Block Forms for Varying Mach Number, Reynolds Number and Yaw Angle," Sandia Corp. Report, SC-4202(TR).

2. Lamb, H., Hydrodynamics, 6th edition, Dover, New York, pp. 102-3.

3. McNown, J. S. and Keulegan, G. H., "Vortex Formation and Resistance in Periodic Motion," Journal of the Engineering Mechanics Division, ASCE, January, 1959.

4. Ludloff, H. F., "On Aerodynamics of Blasts," Advances in Applied Mechanics, Volume III, Academic Press, New York, pp. 109-144.

5. Nøkkentved, Chr., "Variation of the Wind-Pressure Distribution on Sharp-edged Bodies," Laboratorium for Bygningasstatik, Denmark Tekniske Højskole, 1936. and SC-4204(TR)

6. Roshko, A., "On the Drag and Shedding Frequency of Two-Dimensional Bluff Bodies," Washington, NACA TN 3169, July, 1954.

7. Rosenhead, L., "Vortex Systems in Wakes," Advances in Applied Mechanics, Volume III, Academic Press, New York, pp. 185-195.

THE FACTOR OF SAFETY IN DESIGN OF TIMBER STRUCTURES[a]

Discussions by E. George Stern and Richard G. Kimbell, Jr.

E. GEORGE STERN,[1] M. ASCE.—The excellent discussion of safety factors is of great value, since it promotes the proper use of wood as a result of a better understanding of wood.

During the recent testing of full-size structural components, concern was aroused as to the effectiveness of grading rules with respect to boxed-pith lumber. Perfectly satisfactorily looking members were found to incorporate overgrown knots and pin knots, which reduced the effective cross-section of the members excessively and resulted in unexpected wood failures. Had these natural strength-reducing characteristics been along the lumber surface, the members would not have been accepted for the particular grade involved. However, overgrown as they were, these knots were not visible. Maybe, it is desirable to give special consideration to boxed-pith lumber.

RICHARD G. KIMBELL, JR.,[2] M. ASCE.—Mr. Wood is to be complimented on the valuable contribution he has made in presenting a realistic approach to the factor of safety for timber construction. The importance of a realistic evaluation of factor of safety in design cannot be underestimated and it is gratifying that more attention is being given by the profession to what in the past has often been an arbitrarily assumed value.

Although this concept of safety as related to the probability of occurrence of factors tending to cause failure is not new, this paper is the first application to timber design. The use of this procedure should result in a better balanced and more economical design of timber structures. This same philosophy can be applied as well to structures of other materials using factors applicable to these materials.

The author refers to the term "factor of ignorance" as synonomous with factor of safety. The writer prefers to define factor of safety as an insurance factor to guard against the probability of occurrence of unforeseen phenomena tending to cause failure, which is also in agreement with the concept presented by Mr. Wood.

Mr. Wood brings out two important points: That the factor of safety is multi-valued having a "different value for each structural member affected, not only by the strength of that member, but also by the conditions under which it is used," and that there is always a probability of occurrence of failure, even with a large factor of safety. This premise is supported by a statement

a. Proc. Paper 1838, November, 1958, by Lyman W. Wood.
1. Earle B. Norris Research Prof. of Wood Construction, Virginia Polytechnic Inst., Blacksburg, Va.
2. Asst. Director of Technical Services, West Coast Lumbermen's Assn., Portland, Ore.

by Professor Freudenthal, M. ASCE, "The difference between the 'safe' and the 'unsafe' design is in the degree of risk considered acceptable, not in the delusion that such risk can be completely eliminated."

With the continuing progress in the development of electronic computers and an increasing awareness of the usefulness of statistics as a tool, it becomes practical to express the factor of safety in terms of its most probable value, and more important, to indicate to the designing engineer, the probability of failure.

Mr. Wood's presentation is devoted principally to the factor of safety of individual members. Where structural systems utilize repetitive members spaced at close intervals as joints, rafters and laminated decks, the failure of an individual member would not necessarily result in the failure of the entire structure as there would be a redistribution of loads to adjacent members. As there is a range of growth characteristics within a lumber grade, the improbability of occurrence of a consecutive member of low line pieces should also be considered. For this type of framing system the most probable factor of safety of the individual member is of less importance than the most probable factor of safety of the framing system.

LIGHT WOOD TRUSSES[a]

Discussion by E. George Stern

E. GEORGE STERN,[1] M. ASCE.—The studies performed at the U.S. Forest Products Laboratory were limited, covering only 28 trussed rafters of certain slopes, assembled with plywood gusset plates, resorcinol-resin glue and/or common wire nails. Hence, the conclusions drawn from these tests must be restricted to the cases investigated.

It is stated that well designed and well constructed nailed trusses with a slope of 4 in 12 or greater should give adequate service, although (1) only nailed trussed rafters with a slope of 5 in 12 were investigated and (2) nailed trussed rafters with slopes of 2 in 12, 3 in 12, and 4 in 12 were found to perform extremely satisfactorily by other investigators during their extensive studies. Furthermore, the statement is made that the rigidity and strength of glued joints makes glued trusses particularly suitable for trusses with low slopes. On the other hand, (1) nailed trusses of low slopes were not included in the described tests, (2) the tested glued trussed rafters were found to be much more rigid than required under most severe service conditions, and (3) extremely rigid, nailed trussed rafters with low as well as high rafter slopes have been tested and are in use throughout the country.

It is conceded that the nailed trussed rafters described in the paper were assembled with common wire nails, while the nailed trussed rafters which proved to provide such outstanding service are assembled with improved nails, that is, hardened high-carbon-steel, helically threaded nails or with inserted, nailed-on, bent-over or toothed metal plates of various types as are commercially available today.

The author observed that some trussed rafters failed because of rolling shear developed in the plywood gusset plates. This type of failure can be observed if the plywood is fastened with glue which, of necessity, provides only a surface bond along the contact areas. If, on the other hand, nails are used to fasten the plywood gusset plates firmly, the loads are transmitted through the depth of the plywood and lumber. Then, rolling shear cannot take place. In the first case, the glue was stronger than the plywood. In the latter case, the plywood is strengthened by the nails which fasten the plywood.

Other failures were observed because of the fact that the use of common wire nails resulted in joint failure due to lumber splitting and nail bending. The use of the more slender and stiffer, hardened high-carbon-steel nails would probably have prevented such failures on the basis of experience available with these improved nails.

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a. Proc. Paper 1839, November, 1958, by R. F. Luxford.

1. Earle B. Norris Research Prof. of Wood Const., Virginia Polytechnic Inst., Blacksburg, Va.

It was interesting to note that (1) nails in double shear sustain loads about twice those for nails in single shear at small deflections, that is, at design load, and (2) if one-half of the nails subjected to double shear are driven into one side and one-half into the other, their effectiveness at small slips, that is, at design load, is even more than double that of nails in single shear. These findings are significant, in sofar as some U. S. building codes do not allow nails in double shear to transmit twice the load of nails in single shear under any circumstances, because the effectiveness of nails in double shear beyond the design load increases at a lesser rate than the effectiveness of nails in single shear.

The presented test data are significant since they provide valuable information. They are possibly of greatest value when used as a basis of comparison with data obtained for other trussed rafters in commercial use or under development.

GLUED LAMINATED WOOD CONSTRUCTION IN EUROPE[a]

Corrections

CORRECTIONS—On page 1, in line 4 of the Abstract, change "saveuup" to "save up."

On page 1, in footnote 2, change "Madison, Wis." to Portland, Oreg."

On page 1, it should be noted that footnote 3 is continued on page 4.

On page 3, in the third line of the caption for Fig. 2, the word "foot" should be deleted.

On page 16, under the heading "Addendum I. Roof Beam Designs. Problem." the word "load" should be deleted from the line "Rise of beam."

a. Proc. Paper 1840, November, 1958, by M. L. Selbo and A. C. Knauss.

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DESIGN OF PIER BENT AND RIGID FRAME BY A COMPUTER[a]

Discussion by Glen V. Berg

GLEN V. BERG,[1] A.M. ASCE.—The writer feels compelled to challenge the statement that "It is generally true for any problem that the best method for manual computation is also the best method for automatic computation."

This is, in the writer's opinion, an unwise and even dangerous philosophy. There is an understandable urge to accept it, for it implies that the engineer need not learn anything new to use the computer. But acceptance of this philosophy severely limits the scope of the tasks which can be turned over to the computer.

A case in point is the method of moment distribution. There is no doubt of the effectiveness of moment distribution as an engineering tool. It is well known and widely accepted largely because it is explainable on intuitive grounds without recourse to mathematical arguments, and also because the engineer can see the numbers converge to a solution. But just what does moment distribution accomplish? Basically it is nothing more nor less than a particular iterative process for solving the slope-deflection equations. Why, then, should one require the computer to follow the same path that he follows to obtain a solution? Might it not be better in some problems to program the computer to write the slope-deflection equations and solve them by some standard technique such as Gauss-Seidel iteration or Gauss elimination? Indeed, why not use a matrix formulation of the problem, thereby enabling the same program to analyze an entire class of structures? One should not let his familiarity with one process cause him to overlook other methods that might be even better suited to the computer.

Lest he be misunderstood, the writer does not dispute the author's approach to his particular problem. It is the generalization that is in dispute. The fact that one has reached a goal by a particular route for ten or twenty years does not mean that he should adhere to the same route when new and better equipment becomes available. One does not ride the ferry after the bridge is built. Progress requires changing the route. The engineer must apply the effort needed to overcome inertia and alter his course to suit his new environment.

a. Proc. Paper 1854, November, 1958, by Charles P. C. Tung.
1. Asst. Prof. of Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

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DIRECT DESIGN OF OPTIMUM INDETERMINATE TRUSSES[a]

Discussion by E. I. Fiesenheiser

E. I. FIESENHEISER,[1] M. ASCE.—Although the author has extended the basic principles to several examples, this method of direct design of indeterminate truss members is certainly not new. In fact, it was explained in an article entitled

> "Simplified Design of Indeterminate Truss Members" by Merhyle F. Spotts in Civil Engineering, May 1941.

The writer has been teaching the method to his graduate students for the past ten years. It is surprising that Mr. Spotts' excellent article went unnoticed, as it is not listed in the author's bibliography. It does, however, contain the same basic equations in slightly different form. Mr. Spotts, also, does not claim originality. He refers to the previous work of Pippard in

> "Analysis of Engineering Structures" by Pippard and Baker, Longmans, Green and Company, 1936.

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a. Proc. Paper 1867, December, 1958, by Louis M. Laushey.

1. Prof. and Director, Dept. of Civ. Eng., Illinois Inst. of Technology, Chicago, Ill.

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REVIEW OF LIMIT DESIGN FOR STRUCTURAL CONCRETE[a]

Discussion by George C. Ernst

GEORGE C. ERNST,[1] M. ASCE.—The authors have provided a very necessary summary of the development of limit design for reinforced concrete structures. The section on terminology is especially good in presenting definitions classified into three groups that indicate the area of use. Concerning the various theories reviewed in the paper, the writer feels strongly that Professor A. L. L. Baker's general equations have the broadest application for the control of limit design and also permit a relatively convenient method of obtaining an approximate elastic solution for the working stress range.

Recent experimental and theoretical studies(1,2) have demonstrated that it should be possible to redistribute moments in any manner desired for the design of continuous beams and one-bay, single-story rigid frames when diagonal tension requirements are met. A limited study of haunched members,(2) in which the ultimate moment varied directly with the depth, indicated that the required concentrated plastic rotation at a hinge might be reduced to a negligible value by tapering the members of a two-span continuous beam. More information is needed on the effect of haunched members on required and experimentally developed rotations.

With regard to deflections resulting from various distributions of moments, Figures 1, 2, and 3 show that the concern over working load conditions may be unduly emphasized, although the writer readily admits that considerable theoretical and experimental work is needed in this area. The curves shown in the figures are based upon an idealized M-ϕ relationship having distinct elastic and plastic regions. It is true that the deflection curves are not exactly representative of actual conditions, but nevertheless should illustrate relative relationships. With this in mind, the difference between designing by the present ACI Building Code (ACI 318-56) and by an arbitrarily selected statically admissible distribution of moments does not appear to be particularly significant for continuous beams. In general, it would seem that any statically admissible moment diagram that equalized the critical moments as closely as possible would provide a sound basis for limit design of continuous beams and one-story, single-bay rigid frames.

a. Proc. Paper 1878, December, 1958, by C. W. Yu and Eivind Hognestad.

1. Professor of Civil Engineering and Director of the Engineering Experiment Station, University of Nebraska, Lincoln, Nebraska.

REFERENCES

1. Ernst, G. C., "Moment and Shear Redistribution in Two-Span Continuous Reinforced Concrete Beams," Proceedings of the American Concrete Institute, V. 55, Journal V. 30 - No. 5, November 1958, pp. 573-589.

2. Ernst, G. C., "The Redistribution of Moment and Shear in Continuous Beams and Frames of Reinforced Concrete," CE Thesis, University of Michigan, 1958.

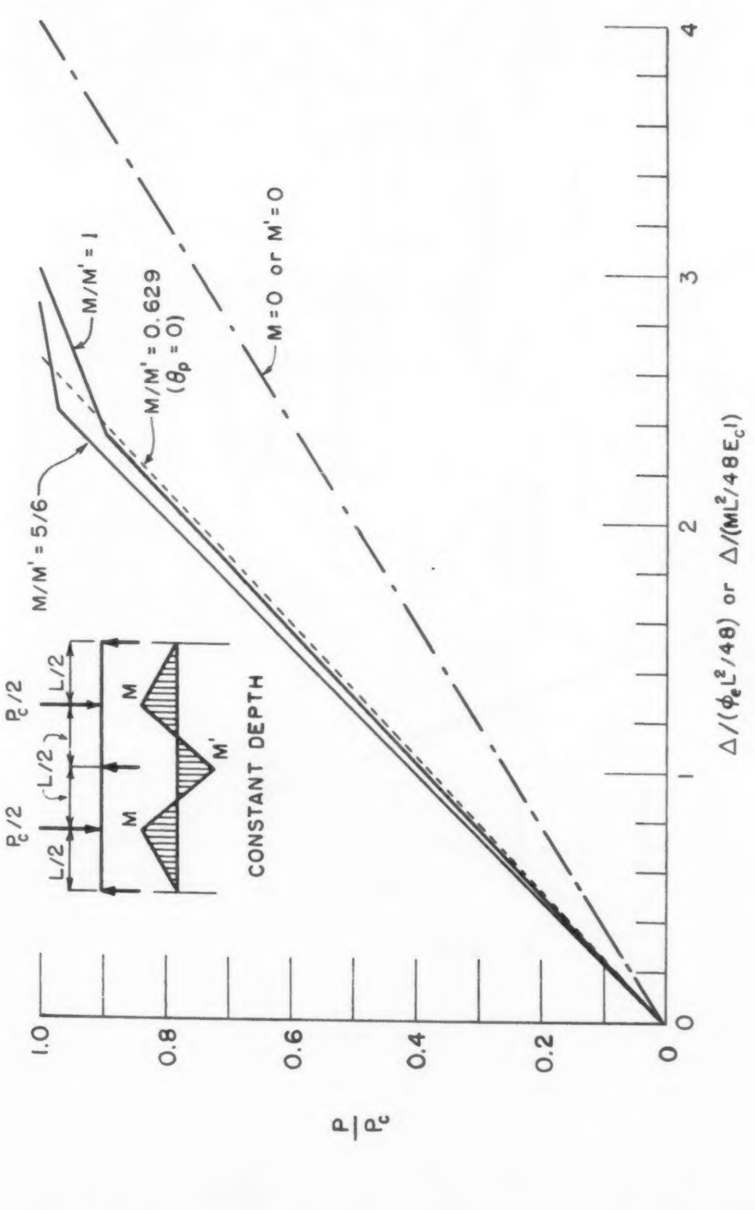
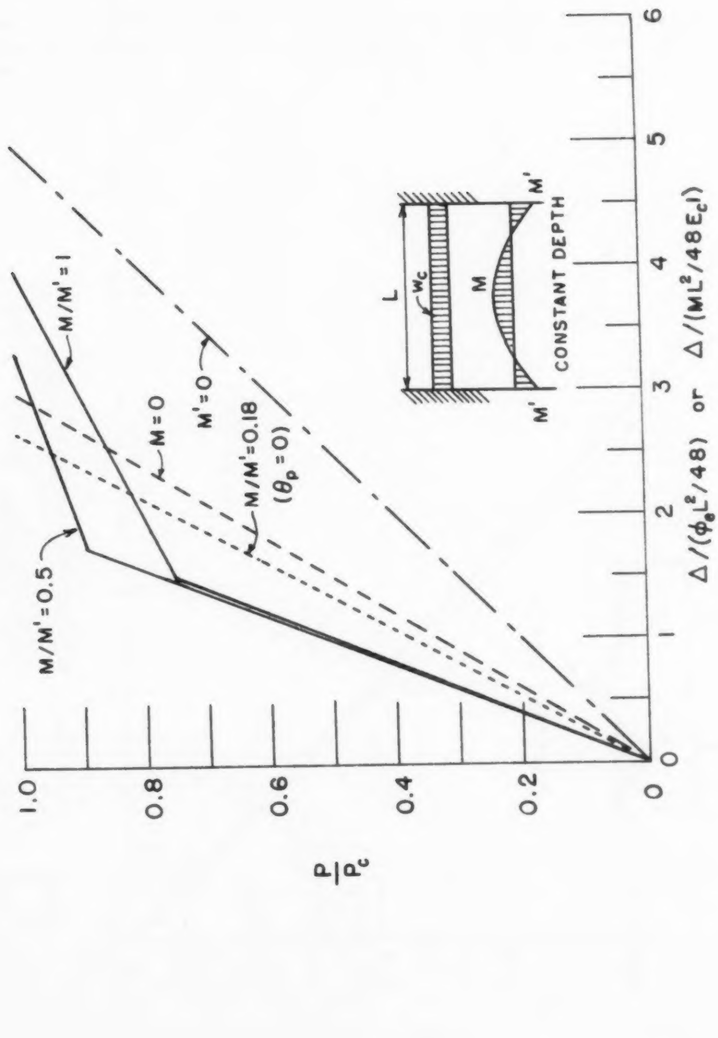
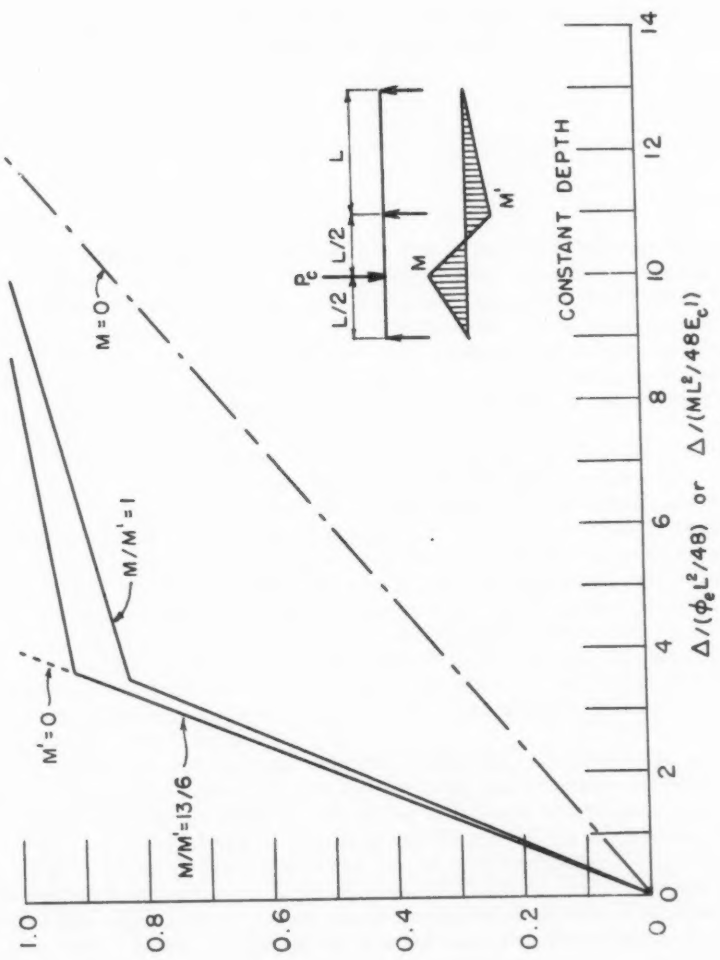

P/P_c
1.0
0.8
0.6
0.4
0.2
0
0
1
2
3
4
Δ/(φ_e L²/48) or Δ/(ML²/48 E_c I)
P_c/2
P_c/2
L/2
L/2
L/2
M
M
M'
CONSTANT DEPTH
M/M' = 5/6
M/M' = 1
M/M' = 0.629
(θ_p = 0)
M = 0 or M' = 0

FIGURE 1



P/P_c
1.0
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
M/M' = 0.5
M/M' = 1
M = 0
M' = 0
M/M' = 0.18
(θ_p = 0)
L
w_c
M
M'
M'
CONSTANT DEPTH
Δ/(φ_e L²/48) or Δ/(ML²/48 E_c I)

FIGURE 2



P/P_c
1.0
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
$\Delta/(\phi_e L^2/48)$ or $\Delta/(ML^2/48E_cI)$
M/M' = 13/6
M' = O
M/M' = 1
M = O
CONSTANT DEPTH
P_c
L/2
L/2
L
M
M'

FIGURE 3

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CORRELATION OF PREDICTED AND OBSERVED SUSPENSION BRIDGE BEHAVIOR[a]

Discussion by F. B. Farquharson

F. B. FARQUHARSON,[1] M. ASCE.—The author is to be commended for his lucid exposition of the validity of dynamic model tests as a means of reliable prediction of prototype behavior. It is indeed fortunate that relatively reliable observations are available on two bridges of record span, one of which was destroyed and the other, (the world's longest), which has oscillated in torsion with such vigor that the bridge was closed to traffic until the storm had abated. It is also significant that the mechanism of the excitation was not the same on the two structures.

Following a trend which became current during the thirties, the stiffening elements which supplemented the cable stiffness assumed increasingly smaller importance, i.e., increasing dependence on the cables and weight for stiffness. Encouraged by the construction of the Golden Gate bridge in 1937, where the truss contributed only about five per cent to the total stiffness, four girder stiffened bridges were built with the stiffness contribution of the girders ranging between about 22 per cent on the Thousand Islands, 20 per cent on the Deer Isle, three per cent on the Bronx Whitestone and 1.5 per cent on the original Tacoma. On all of these bridges, as was the custom of the day, wind was considered only in the calculation of the lateral deflection of the suspended structure. In other words, wind was recognized only as a source of a static force.

It has been mentioned that laboratory investigations have shown that some form of roadway slots are detrimental to the aerodynamic stability of the structure. It should also be pointed out that truss stiffened bridges are very sensitive to certain modifications in the vicinity of the edge of the roadway.

Fig. 1 shows the result of tests on two section models (δ_0 = 0.026) which is a part of the evolution of the final stable Tacoma design. It had previously been determined that the removal of the sidewalk stringers was decidedly beneficial through increasing the positive angle of attack of the wind below which oscillation would not occur. At an angle of attack of $+6^{\circ}$ on a full model of the New Tacoma bridge with a solid deck, the critical wind velocity was increased by 14 per cent by the removal of the sidewalk stringer. (In practice a similar result was achieved by using a latticed stringer.) Motion which previously developed at $\beta = +2^{\circ}$ to $+4^{\circ}$ completely disappeared.(4-d)*

a. Proc. Paper 1944, February, 1959, by George S. Vincent.
1. Prof., Civ. Eng., and Director, Eng. Experiment Station, Univ. of Washington, Seattle, Wash.
*Reference citations are taken from the author's list.

In Fig. 1 double amplitude in degrees of torsional motion is plotted against angle of attack under a fixed wind velocity of 15.5/sec (76.7 mph on prototype). The models used in these tests were constructed in such a manner that all but the deck structure, including the stringers and the top chord, could be removed. Fig. 1 shows that the critical angle of attack was not essentially altered on either model by removing these members. However, the reduction in total air damping has steepened the angle response curves in each case. Quite evidently model (b) shows a gain of about three degrees in the range of positive angles of attack under which the structure was stable.

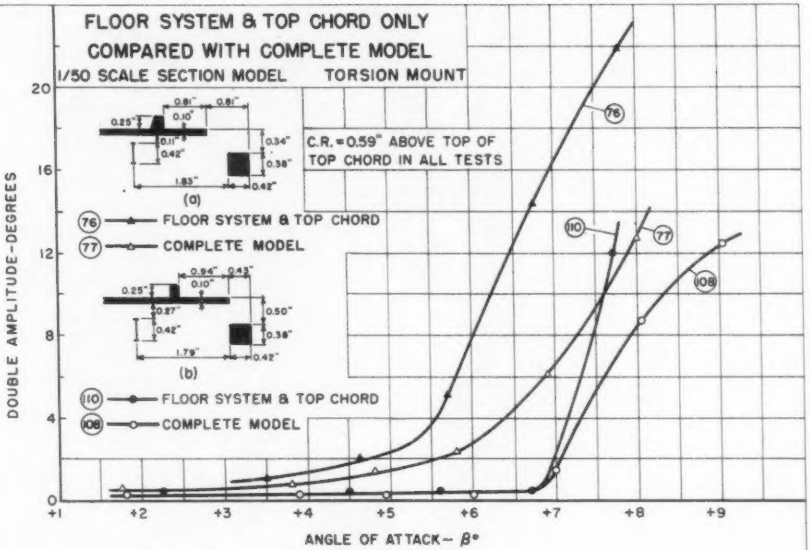
Fig. 2 shows angle response curves developed with three configurations tested on the full model. Curves A and C involved a solid deck with open lattice sidewalk stringers and with two values of torsional stiffness in the suspended structure. In curve B the torsional stiffness was the same as in curve A but with the finally adopted pattern of deck slots.[4-d]

Curves A and C represent a catastrophic response and curve B a strongly restricted response in which at, $\delta_0 = 0.0486$, the maximum double amplitude in torsion never exceeded 1°. Hand rails were not installed on this model, but all investigations into their effect has shown a strong limitation on the amplitude of motion. This effect is a combination of added aerodynamic damping plus a beneficial change in the air flow in the vicinity of the leading edge of the deck.

The curve B response is confined to a narrow velocity range of 6.0 ft/sec to 9.5 ft/sec. The addition of a bottom lateral system would raise the lower critical velocity to 79.5 mph and the higher critical velocity to 125 mph.[2] The addition of the bottom lateral system plus the effect of the hand rails has most certainly increased the damping on the suspended structure by a substantial amount thus eliminating the very meager response shown in Fig. 2.

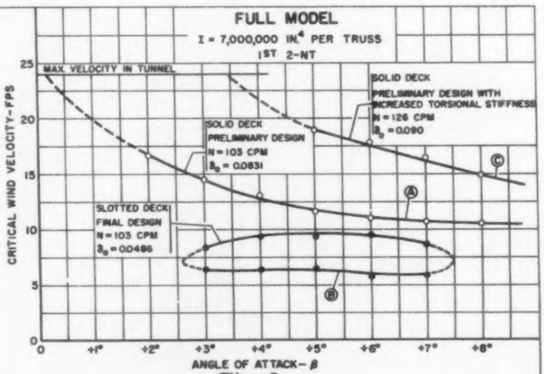
Fig. 3, which shows the site of the Tacoma Narrows bridge, provides an excellent basis for evaluating the effect of the site in modifying the quality of the wind which impinges on the bridge. Northerly winds crossed the bridge from right to left having passed over the peninsula on the right. The record shows that the original bridge rarely oscillated in any significant amount in these winds since they approached at a horizontal angle of roughly 45° and with some positive or negative angle of attack.[4-a]

Southwesterly winds approached over water without obstruction for a distance of about 16 miles and impinged nearly normally on the suspended structure at zero degrees angle of attack and were relatively free of turbulence. The efficiency of southwesterly winds was very much greater than that of northerly winds. In tests on the full model of the original Tacoma Narrows bridge, a wind angled upward at 10° developed 20° of double amplitude in torsion under a wind 55 per cent higher than that required for the same amplitude at an angle of attack of 0°.[4-c]



FLOOR SYSTEM & TOP CHORD ONLY
COMPARED WITH COMPLETE MODEL
1/50 SCALE SECTION MODEL TORSION MOUNT
C.R. = 0.59" ABOVE TOP OF TOP CHORD IN ALL TESTS
(a)
(b)
76 FLOOR SYSTEM & TOP CHORD
77 COMPLETE MODEL
110 FLOOR SYSTEM & TOP CHORD
108 COMPLETE MODEL
DOUBLE AMPLITUDE-DEGREES
ANGLE OF ATTACK– β°

Fig. 1



FULL MODEL
I = 7,000,000 IN.^{4} PER TRUSS
1ST 2-NT
MAX. VELOCITY IN TUNNEL
SOLID DECK
PRELIMINARY DESIGN WITH INCREASED TORSIONAL STIFFNESS
N = 126 CPM
δ_0 = 0.090
SOLID DECK
PRELIMINARY DESIGN
N = 103 CPM
δ_0 = 0.0831
SLOTTED DECK
FINAL DESIGN
N = 103 CPM
δ_0 = 0.0486
CRITICAL WIND VELOCITY-FPS
ANGLE OF ATTACK– β

Fig. 2

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Department of Conditions of Practice are identified by the symbols (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper number are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 1859 is identified as 1859 (HY 7) which indicates that the paper is contained in the seventh issue of the Journal of the Hydraulics Division during 1958.

VOLUME 84 (1958)

FEBRUARY: 1528(HY1), 1529(PO1), 1530(HY1), 1531(HY1), 1532(HY1), 1533(SA1), 1534(SA1), 1535(SM1), 1536(SM1), 1537(SM1), 1538(PO1)[c], 1539(SA1), 1540(SA1), 1541(SA1), 1542(SA1), 1543(SA1), 1544(SM1), 1545(SM1), 1546(SM1), 1547(SM1), 1548(SM1), 1549(SM1), 1550(SM1), 1551(SM1), 1552(SM1), 1553(PO1) 1554(PO1), 1555(PO1), 1556(PO1), 1557(SA1)[c], 1558(HY1)[c], 1559(SM1)[c].

MARCH: 1560(ST2), 1561(ST2), 1562(ST2), 1563(ST2), 1564(ST2), 1565(ST2), 1566(ST2), 1567(ST2). 1568 (WW2), 1569(WW2), 1570(WW2), 1571(WW2), 1572(WW2), 1573(WW2), 1574(PL1), 1575(PL1), 1576(ST2)[c] 1577(PL1), 1578(PL1)[c], 1579(WW2)[c].

APRIL: 1580(EM2), 1581(EM2), 1582(HY2), 1583(HY2), 1584(HY2), 1585(HY2), 1586(HY2), 1587(HY2), 1588 (HY2), 1589(IR2), 1590(IR2), 1591(IR2), 1592(SA2), 1593(SU1), 1594(SU1), 1595(SU1), 1596(EM2), 1597(PO2), 1598(PO2), 1599(PO2), 1600(PO2), 1601(PO2), 1602(PO2), 1603(HY2), 1604(EM2), 1605(SU1)[c], 1606(SA2), 1607(SA2), 1608(SA2), 1609(SA2), 1610(SA2), 1611(SA2), 1612(SA2), 1613(SA2), 1614(SA2)[c], 1615(IR2)[c], 1616 (HY2)[c], 1617(SU1), 1618(PO2)[c], 1619(EM2)[c], 1620(CP1).

MAY: 1621(HW2), 1622(HW2), 1623(HW2), 1624(HW2), 1625(HW2), 1626(HW2), 1627(HW2), 1628(HW2), 1629 (ST3), 1630(ST3), 1631(ST3), 1632(ST3), 1633(ST3), 1634(ST3), 1635(ST3), 1636(ST3), 1637(ST3), 1638(ST3), 1639(WW3), 1640(WW3), 1641(WW3), 1642(WW3), 1643(WW3), 1644(WW3), 1645(SM2), 1646(SM2), 1647 (SM2), 1648(SM2), 1649(SM2), 1650(SM2), 1651(HW2), 1652(HW2)[c], 1653(WW3)[c], 1654(SM2), 1655(SM2), 1656(ST3)[c], 1657(SM2)[c].

JUNE: 1658(AT1), 1659(AT1), 1660(HY3), 1661(HY3), 1662(HY3), 1663(HY3), 1664(HY3), 1665(SA3), 1666 (PL2), 1667(PL2), 1668(PL2), 1669(AT1), 1670(PO3), 1671(PO3), 1672(PO3), 1673(PL2), 1674(PL2), 1675 (PO3), 1676(PO3), 1677(SA3), 1678(SA3), 1679(SA3), 1680(SA3), 1681(SA3), 1682(SA3), 1683(PO3), 1684 (HY3), 1685(SA3), 1686(SA3), 1687(PO3), 1688(SA3)[c], 1689(PO3)[c], 1690(HY3)[c], 1691(PL2)[c].

JULY: 1692(EM3), 1693(EM3), 1694(ST4), 1695(ST4), 1696(ST4), 1697(SU2), 1698(SU2), 1699(SU2), 1700(SU2), 1701(SA4), 1702(SA4), 1703(SA4), 1704(SA4), 1705(SA4), 1706(EM3), 1707(ST4), 1708(ST4), 1709(ST4), 1710 (ST4), 1711(ST4), 1712(ST4), 1713(SU2), 1714(SA4), 1715(SA4), 1716(SU2), 1717(SA4), 1718(EM3), 1719 (EM3), 1720(SU2), 1721(ST4)[c], 1722(ST4), 1723(ST4), 1724(EM3)[c].

AUGUST: 1725(HY4), 1726(HY4), 1727(SM3), 1728(SM3), 1729(SM3), 1730(SM3), 1731(SM3), 1732(SM3), 1733 (PO4), 1734(PO4), 1735(PO4), 1736(PO4), 1737(PO4), 1738(PO4), 1739(PO4), 1740(PO4), 1741(PO4), 1742 (PO4), 1743(PO4), 1744(PO4), 1745(PO4), 1746(PO4), 1747(PO4), 1748(PO4), 1749(PO4).

SEPTEMBER: 1750(IR3), 1751(IR3), 1752(IR3), 1753(IR3), 1754(IR3), 1755(ST5), 1756(ST5), 1757(ST5), 1758(ST5), 1759(ST5), 1760(ST5), 1761(ST5), 1762(ST5), 1763(ST5), 1764(ST5), 1765(WW4), 1766(WW4), 1767(WW4), 1768(WW4), 1769(WW4), 1770(WW4), 1771(WW4), 1772(WW4), 1773(WW4), 1774(IR3), 1775 (IR3), 1776(SA5), 1777(SA5), 1778(SA5), 1779(SA5), 1780(SA5), 1781(WW4), 1782(SA5), 1783(SA5), 1784 (IR3)[c], 1785(WW4)[c], 1786(SA5)[c], 1787(ST5)[c], 1788(IR3), 1789(WW4).

OCTOBER: 1790(EM4), 1791(EM4), 1792(EM4), 1793(EM4), 1794(EM4), 1795(HW3), 1796(HW3), 1797(HW3), 1798(HW3), 1799(HW3), 1800(HW3), 1801(HW3), 1802(HW3), 1803(HW3), 1804(HW3), 1805(HW3), 1806 (HY5), 1807(HY5), 1808(HY5), 1809(HY5), 1810(HY5), 1811(HY5), 1812(SM4), 1813(SM4), 1814(ST6), 1815 (ST6), 1816(ST6), 1817(ST6), 1818(ST6), 1819(ST6), 1820(ST6), 1821(ST6), 1822(EM4), 1823(PO5), 1824 (SM4), 1825(SM4), 1826(SM4), 1827(ST6)[c], 1828(SM4)[c], 1829(HW3)[c], 1830(PO5)[c], 1831(EM4)[c], 1832(HY5)[c]

NOVEMBER: 1833(HY6), 1834(HY6), 1835(SA6), 1836(ST7), 1837(ST7), 1838(ST7), 1839(ST7), 1840(ST7), 1841(ST7), 1842(SU3), 1843(SU3), 1844(SU3), 1845(SU3), 1846(SU3), 1847(SA6), 1848(SA6), 1849(SA6), 1850(SA6), 1851(SA6), 1852(SA6), 1853(SA6), 1854(ST7), 1855(SA6)[c], 1856(HY6)[c], 1857(ST7)[c], 1858 (SU3)[c].

DECEMBER: 1859(HY7), 1860(IR4), 1861(IR4), 1862(IR4), 1863(SM5), 1864(SM5), 1865(ST8), 1866(ST8), 1867 (ST8), 1868(PP1), 1869(PP1), 1870(PP1), 1871(PP1), 1872(PP1), 1873(WW5), 1874(WW5), 1875(WW5), 1876 (WW5), 1877(CP2), 1878(ST8), 1879(ST8), 1880(HY7)[c], 1881(SM5)[c], 1882(ST8)[c], 1883(PP1)[c], 1884(WW5)[c], 1885(CP2)[c], 1886(PO6), 1887(PO6), 1888(PO6), 1889(PO6), 1890(HY7), 1891(PP1).

VOLUME 85 (1959)

JANUARY: 1892(AT1), 1893(AT1), 1894(EM1), 1895(EM1), 1896(EM1), 1897(EM1), 1898(EM1), 1899(HW1), 1900(HW1), 1901(HY1), 1902(HY1), 1903(HY1), 1904(HY1), 1905(PL1), 1906(PL1), 1907(PL1), 1908(PL1), 1909(ST1), 1910(ST1), 1911(ST1), 1912(ST1), 1913(ST1), 1914(ST1), 1915(ST1), 1916(AT1)[c], 1917(EM1)[c], 1918(HW1)[c], 1919(HY1)[c], 1920(PL1)[c], 1921(SA1)[c], 1922(ST1)[c], 1923(EM1), 1924(HW1), 1925(HW1), 1926 (PL1), 1927(HW1), 1928(HW1), 1929(SA1). 1930(SA1), 1931(SA1), 1932(SA1).

FEBRUARY: 1933(HY2), 1934(HY2), 1935(HY2), 1936(SM1), 1937(SM1), 1938(ST2), 1939(ST2), 1940(ST2), 1941(ST2), 1942(ST2), 1943(ST2), 1944(ST2), 1945(HY2), 1946(PO1), 1947(PO1), 1948(PO1), 1949(PO1), 1950(HY2)[c], 1951(SM1)[c], 1952(ST2)[c], 1953(PO1)[c], 1954(CO1), 1955(CO1), 1956(CO1), 1957(CO1), 1958 (CO1), 1959(CO1).

c. Discussion of several papers, grouped by divisions.